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RESEARCH REPORT

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# MODIFIED POWER DIVERGENCE ESTIMATORS: PERFORMANCES IN LOCATION MODELS

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# 1 INTRODUCTION

As was already mentioned in many previous publications, e.g. in Liese & Vajda ([6]), the well known information-theoretic measures of divergence of probability measures introduced in the 60ties by A. Renyi and I. Csiszar cannot be directly applied in statistical estimation, since the divergence between the theoretical absolutely continuous probability measure and the discrete empirical probability measure takes on infinite values.

In 2006 Liese & Vajda ([6]) and independently Broniatowski & Keziou ([1]) proposed two different modifications of divergences, which can be used for minimum divergence estimation. These modification were in 2008 - 2009 studied and extended by Broniatowski & Vajda in [3], who called them *subdivergence* and *superdivergence*.

We follow up their work and focus on important special classes of the so-called *power subdivergences*, *power superdivergences*, and the corresponding estimators. We describe the basic formulas for, and relationships between these estimators, the influence functions, asymptotic variances, and also their actual performances in extensive simulation studies.

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# 2 BASIC CONCEPTS

This chapter introduces the $\phi$-divergences, their basic characteristics, and the concept of minimum divergence estimation. We mention several problems encountered when working with these estimators, and in the third section we study the ways how to bypass them.

## 2.1 $\phi$-divergences

Let $(\mathcal{X}, \mathcal{A})$ be a measurable space and let $\mathcal{P}$ be a set of all probability measures on $(\mathcal{X}, \mathcal{A})$. If $P \in \mathcal{P}$ is dominated by a $\sigma$-finite measure $\lambda$ on $(\mathcal{X}, \mathcal{A})$, then $p = dP/d\lambda$ denotes a Radon-Nikodym density of $P$ with respect to measure $\lambda$.

**Definition 1.** *Let $P, Q \in \mathcal{P}$, $\{P, Q\} \ll \lambda$, $p = dP/d\lambda$ and $q = dQ/d\lambda$. A $\phi$-divergence of distributions $P$ a $Q$ is a function $D_\phi : \mathcal{P} \times \mathcal{P} \to [0, \infty]$ defined by*

$$D_\phi(P, Q) = \int_{\mathcal{X}} \phi\left(\frac{p}{q}\right) dQ = \int_{\mathcal{X}} q\, \phi\left(\frac{p}{q}\right) d\lambda \ , \tag{1}$$

*where $\phi : (0, \infty) \to \mathbb{R}$ is a convex function called generating function.. In (1) we put*

$$q\, \phi\left(\frac{p}{q}\right) = \begin{cases} q\, \phi(0) & if\ p = 0 \\ p\, \phi(\infty)/\infty & if\ q = 0, \end{cases}$$

*where $\phi(0) := \lim_{t \to 0_+} \phi(t)$ and $\phi(\infty)/\infty := \lim_{t\to\infty} \frac{\phi(t)}{t}$, while "$0 \cdot \infty = 0$".*

Basic properties are summarized in the following assertions. Their proofs can be found in [8].

**Theorem 1.** *For each generating function $\phi$ it holds*

$$\phi(0) + \phi(\infty)/\infty > 0$$

*and*

$$\phi(1) \le D_\phi(P, Q) \le \phi(0) + \phi(\infty)/\infty \quad \text{for every} \quad P, Q \in \mathcal{P}\,.$$

*Moreover,*

(1) $D_\phi(P, Q) = \phi(1)$ *if* $P = Q$,
(2) *if $\phi$ is strictly convex at 1,* $D_\phi(P, Q) = \phi(1)$ *iff* $P = Q$ ,
(3) $D_\phi(P, Q) = \phi(0) + \phi(\infty)/\infty$ *if* $P \perp Q$ *(i.e. $P, Q$ are singular),*
(4) *if $\phi$ is strictly convex at 1 and* $\phi(0) + \phi(\infty)/\infty < \infty$, $D_\phi(P, Q) = \phi(0) + \phi(\infty)/\infty$ *iff* $P \perp Q$.

The best known $\phi$-divergences are the Shannon information divergence with $\phi(t) = t\ln t - t + 1$, Kullback-Leibler (also called reversed Shannon) information divergence with $\phi(t) = -\ln t + t - 1$, total variation with $\phi(t) = |t-1|$, Pearson $\chi^2$-divergence with $\phi(t) = (1-t)^2$, and Hellinger divergence with $\phi(t) = (1-\sqrt{t})^2$.

¿From now on, we restrict ourselves to the twice differentiable, strictly convex generating functions $\phi$ with $\phi(1) = 0$ and continuous extension to $t = 0_+$ denoted by $\phi(0)$. We denote by $\Phi$ the class of all such functions. Further, we deal with $P$ and $Q$ which are either measure-theoretically equivalent, $P \equiv Q$ (i.e. $pq > 0$ $\lambda$-a.s.), or measure-theoretically orthogonal, $P \perp Q$ (i.e. $pq = 0$ $\lambda$-a.s.).

In the sequel, we shall use the $\phi_\alpha$-divergences

$$D_\alpha(P,Q) := D_{\phi_\alpha}(P,Q), \quad \alpha \in \mathbb{R}, \tag{2}$$

called *power divergences* defined by the functions

$$\phi_\alpha(t) = \frac{t^\alpha - \alpha(t-1) - 1}{\alpha(\alpha-1)} \qquad \alpha \neq 0, \alpha \neq 1 \tag{3}$$

with the limiting cases

$$\phi_0(t) = -\ln t + t - 1 \quad \text{and} \quad \phi_1(t) = t\ln t - t + 1. \tag{4}$$

Functions $\phi_\alpha$ satisfy the relations

$$\phi_\alpha(0) = \begin{cases} \frac{1}{\alpha} & \text{if} \quad \alpha > 0 \\ \infty & \text{if} \quad \alpha \leq 0 \end{cases} \quad \text{and} \quad \phi_\alpha(\infty)/\infty = \begin{cases} \frac{1}{1-\alpha} & \text{if} \quad \alpha < 1 \\ \infty & \text{if} \quad \alpha \geq 1\,, \end{cases}$$

and therefore

$$\phi_\alpha(0) + \phi_\alpha(\infty)/\infty = \begin{cases} \frac{1}{\alpha(1-\alpha)} & \text{if} \quad 0 < \alpha < 1 \\ \infty & \text{otherwise.} \end{cases}$$

Since $\phi_\alpha(1) = 0$, the $\phi_\alpha$-divergences are given by the formula

$$D_\alpha(P,Q) = \begin{cases} \frac{1}{\alpha(\alpha-1)} \int \left( \left(\frac{p}{q}\right)^\alpha - \alpha\frac{p}{q} + \alpha - 1 \right) dQ = \frac{1}{\alpha(\alpha-1)} \left( \int p^\alpha q^{1-\alpha}\, d\lambda - 1 \right) & \alpha \neq 0, \alpha \neq 1 \\ \int \ln\frac{q}{p}\, dQ & \alpha = 0 \\ \int \ln\frac{p}{q}\, dP & \alpha = 1 \end{cases} \tag{5}$$

and Theorem 1 implies for them

$$0 \leq D_\alpha(P,Q) \leq \begin{cases} \frac{1}{\alpha(1-\alpha)} & \text{if} \quad 0 < \alpha < 1 \\ \infty & \text{otherwise.} \end{cases} \tag{6}$$

The left equality takes place if and only if $P = Q$. If $0 < \alpha < 1$ then the right equality takes place if and only if $P \perp Q$. Otherwise it takes place if $\alpha \leq 0$ and $Q \not\ll P$ or if $\alpha \geq 1$ and $P \not\ll Q$.

Since $\phi_\alpha^*(t) = \phi_{1-\alpha}(t)$ for every $\alpha \in \mathbb{R}$, it follows that

$$D_\alpha(P, Q) = D_{1-\alpha}(Q, P),$$

and that the only symmetric power divergence is $D_{\frac{1}{2}}(P, Q)$. The symmetrized divergences

$$D_{\phi_\alpha + \phi_\alpha^*}(P, Q) = D_\alpha(P, Q) + D_{1-\alpha}(P, Q)$$

are reflexive and symmetric on $\mathcal{P}$.

The above mentioned Kullback-Leibler, Shannon, Pearson, and Hellinger divergences are (up to a constant in some cases) special cases of the previously introduced power divergences for the values of $\alpha$ equal to 0, 1, 2 and 1/2 respectively.

## 2.2 Minimum $\phi$-divergence estimators

Let $X_1, X_2, ..., X_n$ be independent and identically distributed observations governed by $P_{\theta_0} \in \mathcal{P}$, where $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$ is a family of probability measures on $(\mathcal{X}, \mathcal{A})$, $\Theta \subset \mathbb{R}^d$ is a parameter space, and we assume that for every $\theta, \theta_0 \in \Theta$, $\theta \neq \theta_0$ holds $P_\theta \neq P_{\theta_0}$ and $P_\theta \equiv P_{\theta_0}$. Moreover, we assume the family $\mathcal{P}$ to be nonatomic (continous), i.e. for all $\theta \in \Theta$ and $x \in \mathcal{X}$ we require $P_\theta(\{x\}) = 0$.

The data $X_1, X_2, ..., X_n$ can be represented by empirical probability measures $P_n = \frac{1}{n}\sum_{i=1}^n P_{X_i}$, where $P_x$ is the Dirac probability measure with all mass concenrated at the point $x \in \mathcal{X}$.

**Definition 2.** *We say that a sequence of mappings $\widehat{\theta}_n : \mathcal{X}^n \to \Theta$ is an estimator of parameter $\theta_0 \in \Theta$, if for every $n \in \mathbb{N}$ the mapping $\widehat{\theta}_n$ is measurable and $P_{\widehat{\theta}_n(\mathbf{x})} \in \mathcal{P}$ for every $\mathbf{x} \in \mathcal{X}^n$. The mappings $\widehat{\theta}_n$ themselves are called estimates of $\theta_0 \in \Theta$.*

As we have already shown, for $\phi \in \Phi$ it holds $D_\phi(P_\theta, P_{\theta_0}) \geq 0$ for every $\theta$, $\theta_0 \in \Theta$ with the equality iff $\theta = \theta_0$. Hence, the parameter $\theta_0$ is the unique minimizer of the $\phi$-divergence $D_\phi$, and since we know that the empirical probability measures $P_n$ converge weakly to $P_{\theta_0}$ as $n \to \infty$, it is reasonable to define the minimum $\phi$-divergence estimator as follows.

**Definition 3.** *Let $\phi \in \Phi$. We say that an estimator $\widehat{\theta}_n : \mathcal{X}^n \to \Theta$ of a true parameter $\theta_0 \in \Theta$ is a minimum $\phi$-divergence estimator if for the corresponding $D_\phi$*

$$\widehat{\theta}_n = \text{argmin}_\theta D_\phi(P_\theta, P_n).$$

We face here the problem that the continuous family $\mathcal{P}$ and the family of empirical distributions $\mathcal{P}_{emp}$ are measure-theoretically orthogonal, i.e. $P_\theta \perp P_n$ for every $P_\theta \in \mathcal{P}$ and $P_n \in \mathcal{P}_{emp}$. This means that for every $P_\theta \in \mathcal{P}$ and $P_n \in \mathcal{P}_{emp}$

$$D_\phi(P_\theta, P_n) = \phi_\alpha(0) + \phi_\alpha(\infty)/\infty$$

so that the above defined estimates are trivial.

To bypass this problem, it is possible to use some prior smoothing of the data, or to use a nonparametric density estimation as we did by implementing histogram in [4], but these methods generate other unpleasant problems such as bandwidth selection.

In the next section, we give several modifications of the minimum divergence rule to avoid these complications. Minimum superdivergence estimators were proposed independently by Liese & Vajda in 2006 ([6]) as "modified $\phi$-divergence estimators" and Broniatowski & Keziou in 2006 ([1]) as "minimum dual $\phi$-divergence estimators". Maximum subdivergence estimators were in 2009 introduced by Broniatowski & Keziou ([2]) under the name "dual $\phi$-divergence estimators".

### 2.3 Power subdivergence and power superdivergence estimators

Throughout the next chapters are presented the results of Vajda ([3]) which are then the objects of our further research, most frequently we apply them or verify them in computer simulations.

We consider the probability measures $P \in \mathcal{P}$ and $Q \in \mathcal{Q}$ for $\mathcal{Q} = \mathcal{P} \cup \mathcal{P}_{emp}$. Measures $P, Q$ are either measure-theoretically equivalent (if $Q \in \mathcal{P}$) or measure-theoretically orthogonal (if $Q \in \mathcal{P}_{emp}$). Therefore, the $\phi$-divergences $D_\phi(P, Q)$ are well defined by (1) for all pairs $(P, Q) \in \mathcal{P} \otimes \mathcal{Q}$.

Consider the functionals of $P_\theta, Q$ defined by

$$\underline{\mathrm{D}}_{\phi,\tilde{\theta}}\,(P_\theta, Q) = \int \phi'(p_\theta/p_{\tilde{\theta}})\,\mathrm{d}P_\theta + \int \phi^{\#}(p_\theta/p_{\tilde{\theta}})\,\mathrm{d}Q, \quad (P_\theta,\ Q) \in \mathcal{P} \otimes \mathcal{Q} \tag{7}$$

and parametrized by $(\phi, \tilde{\theta}) \in \Phi \otimes \Theta$, where

$$\phi^{\#}(t) = \phi(t) - t\phi'(t) \quad \text{for every} \quad \phi \in \Phi \tag{8}$$

and $\phi'$ denotes the derivative of $\phi$. For (7) to be correctly defined, we assume that the integrals exist and have a finite value.

The *maximum subdivergence estimators* (briefly, the max$\underline{\mathrm{D}}_\phi$-estimators) with escort parameter $\theta \in \Theta$ are defined as

$$\tilde{\theta}_{\phi,\theta,n} = \mathrm{argmax}_{\tilde{\theta}}\ \underline{\mathrm{D}}_{\phi,\tilde{\theta}}(P_\theta, P_n) \tag{9}$$

$$= \mathrm{argmax}_{\tilde{\theta}} \left[ \int \phi'\left(\frac{p_\theta}{p_{\tilde{\theta}}}\right) \mathrm{d}P_\theta + \frac{1}{n}\sum_{i=1}^{n} \phi^{\#}\left(\frac{p_\theta(X_i)}{p_{\tilde{\theta}}(X_i)}\right)\right] \tag{10}$$

and the *minimum superdivergence estimators* (briefly, the min $\bar{\mathrm{D}}_\phi$-estimators) as

$$\theta_{\phi,n} = \mathrm{argmin}_\theta\ \sup_{\tilde{\theta}}\ \underline{\mathrm{D}}_{\phi,\tilde{\theta}}(P_\theta, P_n) \tag{11}$$

$$= \mathrm{argmin}_\theta\ \sup_{\tilde{\theta}} \left[ \int \phi'\left(\frac{p_\theta}{p_{\tilde{\theta}}}\right) \mathrm{d}P_\theta + \frac{1}{n}\sum_{i=1}^{n} \phi^{\#}\left(\frac{p_\theta(X_i)}{p_{\tilde{\theta}}(X_i)}\right)\right]. \tag{12}$$

According to [3], both the maximum subdivergence and minimum superdivergence estimators are Fisher consistent.

If we restrict ourselves to a subclass of these estimators determined by the power divergences given in (2), by employing the power functions $\phi_\alpha$ from (3) and (4) we receive for $\alpha > 0$ formulas

$$\tilde{\theta}_{\alpha,\theta,n} = \text{argmin}_{\tilde{\theta}} \; M_{\alpha,\theta}(P_n, \tilde{\theta}) \tag{13}$$

and

$$\theta_{\alpha,n} = \text{argmax}_{\theta} \text{inf}_{\tilde{\theta}} \; M_{\alpha,\theta}(P_n, \tilde{\theta}) \;\; \text{or} \;\; \theta_{\alpha,n} = \text{argmax}_{\theta} \; M_{\alpha,\theta}(P_n, \tilde{\theta}_{\alpha,\theta,n}) \tag{14}$$

where

$$\begin{aligned} M_{\alpha,\theta}(P_n, \tilde{\theta}) &= \frac{1}{1-\alpha} \int \left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha \mathrm{d}P_{\tilde{\theta}} + \frac{1}{\alpha n} \sum_{i=1}^n \left(\frac{p_\theta(X_i)}{p_{\tilde{\theta}}(X_i)}\right)^\alpha \quad \text{if } \alpha > 0,\ \alpha \neq 1 \\ &= -\int \ln \frac{p_\theta}{p_{\tilde{\theta}}} \, \mathrm{d}P_\theta + \frac{1}{n} \sum_{i=1}^n \frac{p_\theta(X_i)}{p_{\tilde{\theta}}(X_i)} \quad \text{if } \alpha = 1 \end{aligned} \tag{15}$$

for all $Q \in \mathcal{Q} = \mathcal{P} \cup \mathcal{Q}$ and

$$\tilde{\theta}_{0,\theta,n} = \text{argmax}_{\tilde{\theta}} \; \Sigma_{i=1}^n \ln p_{\tilde{\theta}}(X_i) \;\; \text{and} \;\; \theta_{0,n} = \text{argmax}_\theta \; \Sigma_{i=1}^n \ln p_\theta(X_i) \tag{16}$$

for $\alpha = 0$. It is obvious that in this case the estimators coincide with MLE's, hence the classes of $\max \underline{\mathrm{D}}_\phi$-estimators and of $\min \bar{\mathrm{D}}_\phi$-estimators are extensions of the MLE.

## 2.4 Power subdivergence estimators and power superdivergence estimators in the normal distribution model

Let the observation space $(\mathcal{X}, \mathcal{A})$ be $(\mathbb{R}, \mathcal{B})$ and let $\mathcal{P} = \{P_{\mu,\sigma} : \mu \in \mathbb{R},\ \sigma > 0\}$ be the normal family with parameters of location $\mu$ and scale $\sigma$ (i.e. variances $\sigma^2$). We are interested in the $\max \underline{\mathrm{D}}_\alpha$-estimates $(\tilde{\mu}_{\alpha,\mu,\sigma,n}, \tilde{\sigma}_{\alpha,\mu,\sigma,n})$ with power parameters $\alpha \geq 0$ and escort parameters $(\mu, \sigma) \in \mathbb{R} \otimes (0, \infty)\}$.

If $\alpha = 0$ then these estimators reduce to

$$(\tilde{\mu}_{0,\mu,\sigma,n}, \tilde{\sigma}_{0,\mu,\sigma,n}) = \left( \frac{1}{n} \sum_{i=1}^n X_i,\ \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \tilde{\mu}_{0,n})^2} \right) \tag{17}$$

which is a maximum likelihood estimate in the family of normal distributions.

For $\alpha > 0$, $\alpha \neq 1$ the function (15) becomes

$$M_{\alpha,\mu,\sigma}(P_n, \tilde{\mu}, \tilde{\sigma}) = \frac{1}{1-\alpha} \int \left(\frac{p_{\mu,\sigma}}{p_{\tilde{\mu},\tilde{\sigma}}}\right)^\alpha \mathrm{d}P_{\tilde{\mu},\tilde{\sigma}} + \frac{1}{\alpha n} \sum_{i=1}^n \left(\frac{p_{\mu,\sigma}(X_i)}{p_{\tilde{\mu},\tilde{\sigma}}(X_i)}\right)^\alpha \tag{18}$$

where

$$\left(\frac{p_{\mu,\sigma}(x)}{p_{\tilde{\mu},\tilde{\sigma}}(x)}\right)^\alpha = \left(\frac{\tilde{\sigma}}{\sigma}\right)^\alpha \exp\left\{ \frac{\alpha\,(x-\tilde{\mu})^2}{2\tilde{\sigma}^2} - \frac{\alpha\,(x-\mu)^2}{2\sigma^2} \right\}, \tag{19}$$

and

$$\int \left(\frac{p_{\mu,\sigma}}{p_{\tilde{\mu},\tilde{\sigma}}}\right)^{\alpha} \mathrm{d}P_{\tilde{\mu},\tilde{\sigma}} = \exp\left\{\frac{-\alpha(1-\alpha)(\mu-\tilde{\mu})^2}{2[\alpha\tilde{\sigma}^2+(1-\alpha)\sigma^2]} - \ln\frac{\sqrt{\alpha\tilde{\sigma}^2+(1-\alpha)\sigma^2}}{\tilde{\sigma}^{\alpha}\sigma^{1-\alpha}}\right\}. \tag{20}$$

For $\alpha = 1$

$$\begin{aligned} M_{1,\mu,\sigma}(P_n,\tilde{\mu},\tilde{\sigma}) &= \lim_{\alpha\to 1} M_{\alpha,\mu,\sigma}(P_n,\tilde{\mu},\tilde{\sigma}) \\ &= \frac{-(\mu-\tilde{\mu})^2}{2\tilde{\sigma}^2} - \frac{1}{2}\left[-\ln\left(\frac{\sigma}{\tilde{\sigma}}\right)^2 + \left(\frac{\sigma}{\tilde{\sigma}}\right)^2 - 1\right] \\ &\quad + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\tilde{\sigma}}{\sigma}\right)\exp\left\{\frac{(X_i-\tilde{\mu})^2}{2\tilde{\sigma}^2} - \frac{(X_i-\mu)^2}{2\sigma^2}\right\}. \end{aligned} \tag{21}$$

**Power subdivergence estimators of location:** Let $\mathcal{P} = \{P_\mu : \mu \in \mathbb{R}\}$ be the standard normal family with the location parameter $\mu$ and scale $\sigma = 1$. Then the function (18) for $\alpha > 0, \alpha \neq 1$ takes the form of

$$M_{\alpha,\mu}(P_n,\tilde{\mu}) = \frac{1}{1-\alpha}\left(\eta_{\alpha,\mu}(\mu,\tilde{\mu})\right)^{\alpha-1} + \frac{1}{\alpha n}\sum_{i=1}^{n}\eta_{\alpha,\mu}(X_i,\tilde{\mu}) \tag{22}$$

where

$$\eta_{\alpha,\mu}(x,\tilde{\mu}) = \exp\left\{\alpha(\tilde{\mu}-\mu)(\tilde{\mu}+\mu-2x)/2\right\}, \qquad x \in \mathbb{R}.$$

The max$\underline{\mathrm{D}}_\alpha$-estimates $\tilde{\mu}_{\alpha,\mu,n}$ of location parameter $\mu_0$ with the divergence parameters $\alpha \geq 0$ and escort parameters $\mu \in \mathbb{R}$ are

$$\tilde{\mu}_{0,\mu,n} = \bar{\boldsymbol{X}}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad \text{(MLE)} \tag{23}$$

for $\alpha = 0$ and for $\alpha > 0$ the estimators are

$$\tilde{\mu}_{\alpha,\mu,n} = \mathrm{argmin}_{\tilde{\mu}} M_{\alpha,\mu}(P_n,\tilde{\mu}). \tag{24}$$

In [3], Vajda shows that the estimators (24) are Fisher consistent in the normal family $\mathcal{P}_\sigma = \{P_{\mu,\sigma} = N(\mu,\sigma^2) : \mu \in \mathbb{R}\}$ with $\sigma > 0$ fixed if and only if $\sigma = 1$, which suggests an easy loss of consistency of these estimators. We shall inspect this property by simulations in the next chapters. We shall also examine whether the max$\underline{\mathrm{D}}_\alpha$-estimators escorted by MLE $\tau_n = \tilde{\mu}_{0,\mu,n}$, i.e. $\tilde{\mu}_{\alpha,\tau_n,n}$, are Fischer consistent under all hypothetical models $P_{\mu,\sigma} = N(\mu,\sigma^2)$, $\sigma > 0$, and possibly consistent and robust under the contaminated versions of these models.

**Power superdivergence estimators of location:** Formulas for the min$\bar{\mathrm{D}}_\phi$-estimators of normal location can be derived easily from the previous.

## 3 INFLUENCE FUNCTION AND ASYMPTOTIC VARIANCE

To study the robusness of the previously introduced estimators, we shall use the basic definitions and theorems of robust statistics ([5]) and the influence functions derived in Vajda ([3]). We apply them to obtain a general formula for asymptotic variance.

Lets consider the Dirac probability measures $\delta_x$ on $(\mathcal{X}, \mathcal{A})$ and the convex mixtures

$$Q_{x,\varepsilon} = (1-\varepsilon)Q + \varepsilon\delta_x \quad \text{for} \quad Q \in \mathcal{Q}, \ \ x \in \mathcal{X} \text{ and } 0 \le \varepsilon < 1. \tag{25}$$

We assume $\Theta \subset \mathbb{R}^d$ and consider functions $\psi : \mathcal{X} \times \Theta \mapsto \mathbb{R}^d$ measurable in $x$ and the related mappings $\Psi : \mathcal{Q} \times \Theta \mapsto \mathbb{R}$ defined by

$$\Psi(Q,\theta) = \int \boldsymbol{\psi}(x,\theta)\, \mathrm{d}Q(x) \quad \text{or} \quad \boldsymbol{\psi}(x,\theta) = \Psi(\delta_x, \theta). \tag{26}$$

**Definition 4.** *A functional defined by means of a function $\rho : \mathcal{X} \times \Theta \mapsto \mathbb{R}$ as*

$$T(Q) = argmin_\theta \int \rho(\tilde{x},\theta)\, \mathrm{d}Q(\tilde{x}) \tag{27}$$

*or through a function $\psi : \mathcal{X} \times \Theta \mapsto \mathbb{R}^d$ as the solution of equation*

$$\int \psi(\tilde{x},\theta)\, \mathrm{d}Q(\tilde{x}) = 0 \tag{28}$$

*is called an M-estimator.*

**Definition 5.** *Let $\theta_n : \mathcal{P}_{\mathrm{emp}} \mapsto \Theta$ be an estimator of $\theta_0 \in \Theta$ defined by $\theta_n = T(P_n)$ where $T(Q_{x,\varepsilon})$ is an M-estimator and solves the equation $\Psi(Q_{x,\varepsilon},\theta) = 0$ for all mixtures $Q_{x,\varepsilon}$ with sufficiently small $\varepsilon$.*

*If for some $Q \in \mathcal{P}$ the limits*

$$IF(x;T,Q) = \lim_{\varepsilon \downarrow 0} \frac{T(Q_{x,\varepsilon}) - T(Q)}{\varepsilon} \tag{29}$$

*exist for all $x \in \mathcal{X}$ then (29) is called an* influence function *of the estimator $\theta_n$ on $\mathcal{X}$ at $Q$.*

In the following theorem we assume the existence of the matrix functions

$$\mathring{\boldsymbol{\psi}}(x,\theta) = \left(\frac{\mathrm{d}}{\mathrm{d}\theta}\right)^t \boldsymbol{\psi}(x,\theta) \quad \text{on} \quad \mathcal{X} \otimes \Theta \tag{30}$$

and the expectations

$$\boldsymbol{I}(Q) = Q \cdot \mathring{\boldsymbol{\psi}}(x, T(Q)), \quad Q \in \mathcal{P}. \tag{31}$$

**Theorem 2.** *If the influence function (29) exists then it is given by the formula*

$$IF(x;T,Q) = -\boldsymbol{I}(Q)^{-1}\, \boldsymbol{\psi}(x,T(Q)). \tag{32}$$

**Definition 6.** *The estimator $\theta_n = T(P_n)$ is said to be Fisher consistent if*

$$T(P_\theta) = \theta \quad \textit{for all } \theta \in \Theta. \tag{33}$$

The influence function of a Fisher consistent estimator at $Q = P_\theta$ is

$$\mathrm{IF}(x; T, P_\theta) = -\boldsymbol{I}(P_\theta)^{-1}\, \boldsymbol{\psi}(x, \theta). \tag{34}$$

Under suitable regularity conditions on functions $\boldsymbol{\psi}$ and $F$ ([5]), $\sqrt{n}(T(P_n) - T(Q))$ is asymptotically normal with mean 0 and variance

$$V(T, Q) = \int \mathrm{IF}^2(x; T, Q)\, \mathrm{d}Q = \boldsymbol{I}(Q)^{-2} \int \boldsymbol{\psi}^2(x, T(Q))\, \mathrm{d}Q\,. \tag{35}$$

For a Fisher consistent estimator at $Q = P_\theta$ we then receive

$$V(T, P_\theta) = \boldsymbol{I}(P_\theta)^{-2} \int \boldsymbol{\psi}^2(x, \theta)\, \mathrm{d}P_\theta\,. \tag{36}$$

### 3.1 Influence functions and asymptotic variances of power subdivergence estimators

Restrict ourselves to the vector parameter spaces $\Theta \subset \mathbb{R}^d$ and to the densities $p_\theta$ twice differentiable with respect to $\theta$. We suppose that the functions $M_{\alpha,\theta}(Q, \tilde{\theta})$ given by (15) are twice differentiable in the vector variable $\tilde{\theta}$ and that the differentiation is interchangeable with the integration in (10).

In the sequel, we denote

$$s_\theta = \frac{\mathrm{d}}{\mathrm{d}\theta} \ln p_\theta \quad \text{and} \quad \mathring{s}_\theta = \left(\frac{\mathrm{d}}{\mathrm{d}\theta}\right)^{\mathrm{t}} s_\theta \quad \text{with } {}^{\mathrm{t}} \text{ meaning transposition} \tag{37}$$

and we deal with the max$\underline{\mathrm{D}}_\alpha$-estimators $\tilde{\theta}_{\alpha,\theta,n}$ (13) with the power parameters $\alpha > 0$ and escort parameters $\theta \in \Theta$ given in a general;ized form by

$$\tilde{T}_{\alpha,\theta}(Q) = \mathrm{argmin}_{\tilde{\theta}}\, M_{\alpha,\theta}(Q, \tilde{\theta}) \qquad \alpha > 0. \tag{38}$$

In other words, the values $\tilde{T}_{\alpha,\theta}(Q)$ solve the equations $\Psi_{\alpha,\theta}(Q, \tilde{\theta}) = 0$ in variable $\tilde{\theta} \in \Theta$ for the derivatives

$$\Psi_{\alpha,\theta}(Q, \tilde{\theta}) = \frac{\mathrm{d}}{\mathrm{d}\tilde{\theta}}\, M_{\alpha,\theta}(Q, \tilde{\theta}) = \int \left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha s_{\tilde{\theta}}\, \mathrm{d}P_{\tilde{\theta}} - \int \left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha s_{\tilde{\theta}}\, \mathrm{d}Q. \tag{39}$$

Therefore, the estimates $\tilde{\theta}_{\alpha,\theta,n} = \tilde{T}_{\alpha,\theta}(Q)$ are for all $\alpha > 0$ solutions of the equation

$$P_{\tilde{\theta}} \cdot \left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha s_{\tilde{\theta}} - \frac{1}{n} \sum_{i=1}^{n} \left(\frac{p_\theta(X_i)}{p_{\tilde{\theta}}(X_i)}\right)^\alpha s_{\tilde{\theta}}(X_i) = 0. \tag{40}$$

If we denote

$$\psi_{\alpha,\theta}(x, \tilde{\theta}) = \Psi_{\alpha,\theta}(\delta_x, \tilde{\theta}) \quad \text{and} \quad \mathring{\psi}_{\alpha,\theta}(x, \tilde{\theta}) = \left(\frac{\mathrm{d}}{\mathrm{d}\tilde{\theta}}\right)^{\mathrm{t}} \psi_{\alpha,\theta}(x, \tilde{\theta}) \tag{41}$$

then

$$\mathring{\psi}_{\alpha,\theta}(x,\tilde{\theta}) = \int \left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha s_{\tilde{\theta}}^{\mathrm{t}} s_{\tilde{\theta}} \,\mathrm{d}P_{\tilde{\theta}} - \int \Lambda_{\alpha,\theta,\tilde{\theta}} \,\mathrm{d}P_{\tilde{\theta}} + \Lambda_{\alpha,\theta,\tilde{\theta}}(x)$$

where

$$\Lambda_{\alpha,\theta,\tilde{\theta}}(x) = \left(\frac{p_\theta(x)}{p_{\tilde{\theta}}(x)}\right)^\alpha \left[\alpha s_{\tilde{\theta}}(x)^{\mathrm{t}} s_{\tilde{\theta}}(x) - \mathring{s}_{\tilde{\theta}}(x)\right].$$

Consequently we get

$$\boldsymbol{I}_{\alpha,\theta}(P_{\tilde{\theta}}) = \int \mathring{\psi}_{\alpha,\theta}(\tilde{x},\tilde{\theta}) \,\mathrm{d}P_{\tilde{\theta}}(\tilde{x}) = \int \left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha s_{\tilde{\theta}}^{t} s_{\tilde{\theta}} \,\mathrm{d}P_{\tilde{\theta}} \tag{42}$$

and the influence function of the max$\underset{\sim}{\mathrm{D}}_\phi$-estimators $\tilde{\theta}_{\alpha,\theta,n}$ of the parameter $\theta_0$ with divergence and escort parameters $\alpha > 0$ and $\theta \in \Theta$ is given by the formula

$$\begin{aligned}
\mathrm{IF}(x;\tilde{T}_{\alpha,\theta},P_{\tilde{\theta}}) &= -\boldsymbol{I}_{\alpha,\theta}(P_{\tilde{\theta}})^{-1}\psi_{\alpha,\theta}(x,\tilde{\theta}) \\
&= \boldsymbol{I}_{\alpha,\theta}(P_{\tilde{\theta}})^{-1}\left[\left(\frac{p_\theta(x)}{p_{\tilde{\theta}}(x)}\right)^\alpha s_{\tilde{\theta}}(x) - P_{\tilde{\theta}}\left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha s_{\tilde{\theta}}\right].
\end{aligned} \tag{43}$$

By a basic theorem of robust statistics, for the asymptotic variance we obtain

$$\begin{aligned}
\mathrm{V}(\tilde{T}_{\alpha,\theta},P_{\tilde{\theta}}) &= \boldsymbol{I}_{\alpha,\theta}(P_{\tilde{\theta}})^{-2}\int \psi_{\alpha,\theta}^2(x,\tilde{\theta}) \,\mathrm{d}P_{\tilde{\theta}} \\
&= \boldsymbol{I}_{\alpha,\theta}(P_{\tilde{\theta}})^{-2}\int \left[\left(\frac{p_\theta(x)}{p_{\tilde{\theta}}(x)}\right)^\alpha s_{\tilde{\theta}}(x) - P_{\tilde{\theta}}\left(\frac{p_\theta}{p_{\tilde{\theta}}}\right)^\alpha s_{\tilde{\theta}}\right]^2 \mathrm{d}P_{\tilde{\theta}}.
\end{aligned} \tag{44}$$

If $\alpha = 0$ then we get for all escort parameters $\theta$ the influence function and asymptotic variance of the MLE as follows

$$\mathrm{IF}(x;\tilde{T}_{0,\theta},P_{\tilde{\theta}}) = \boldsymbol{I}(P_{\tilde{\theta}})^{-1}s_{\tilde{\theta}}(x) \quad \text{and} \quad \mathrm{V}(\tilde{T}_{0,\theta},P_{\tilde{\theta}}) = \boldsymbol{I}(P_{\tilde{\theta}})^{-2}\int s_{\tilde{\theta}}^2(x) \,\mathrm{d}P_{\tilde{\theta}} \tag{45}$$

where

$$\boldsymbol{I}(P_{\tilde{\theta}}) = P_{\tilde{\theta}} \cdot s_{\tilde{\theta}}^{\mathrm{t}} s_{\tilde{\theta}}. \tag{46}$$

We get the same influence function and asymptotic variance for all power parameters $\alpha \geq 0$ if the escort parameter $\theta$ coincides with the true parameter $\tilde{\theta}$.

**Influence function and asymptotic variance of power subdivergence estimators of location:** Let $\mathcal{P} = \{P_\mu : \mu \in \mathbb{R}\}$ be the standard normal family with the location parameter $\mu$ and scale $\sigma = 1$. Now the function (39) takes on the form

$$\Psi_{\alpha,\mu}(Q,\tilde{\mu}) = \frac{\mathrm{d}}{\mathrm{d}\tilde{\mu}} M_{\alpha,\mu}(Q,\tilde{\mu}) = Q \cdot (\tilde{\mu} - x)\eta_{\alpha,\mu}(x,\tilde{\mu}) - \alpha(\tilde{\mu} - \mu)\eta_{\alpha,\mu}^{\alpha-1}(\mu,\tilde{\mu}), \tag{47}$$

so that

$$\psi_{\alpha,\mu}(x,\tilde{\mu}) = \Psi_{\alpha,\mu}(\delta_x,\tilde{\mu}) = (\tilde{\mu}-x)\,\eta_{\alpha,\mu}(x,\tilde{\mu}) - \alpha(\tilde{\mu}-\mu)\,\eta_{\alpha,\mu}^{\alpha-1}(\mu,\tilde{\mu}) \tag{48}$$

and

$$\begin{aligned} \mathring{\psi}_{\alpha,\mu}(x,\tilde{\mu}) &= \left(\frac{\mathrm{d}}{\mathrm{d}\tilde{\mu}}\right)\psi_{\alpha,\theta}(x,\tilde{\mu}) \\ &= (1+\alpha(\tilde{\mu}-x)^2)\,\eta_{\alpha,\mu}(x,\tilde{\mu}) - \alpha(1+\alpha(\alpha-1)(\tilde{\mu}-\mu)^2)\,\eta_{\alpha,\mu}^{\alpha-1}(\mu,\tilde{\mu}). \end{aligned} \tag{49}$$

The Fisher information at $P_{\tilde{\mu}}$ is

$$\begin{aligned} \boldsymbol{I}_{\alpha,\mu}(P_{\tilde{\mu}}) &= \int\left(\frac{p_\mu}{p_{\tilde{\mu}}}\right)^\alpha s_{\tilde{\mu}}^2\,\mathrm{d}P_{\tilde{\mu}} = \int\left(\frac{p_\mu}{p_{\tilde{\mu}}}\right)^\alpha (x-\tilde{\mu})^2\,\mathrm{d}P_{\tilde{\mu}} \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(\tilde{\mu}-x)^2 e^{-(\alpha(x-\mu)^2+(1-\alpha)(x-\tilde{\mu})^2)/2}\mathrm{d}x \\ &= \frac{1}{\sqrt{2\pi}}\,e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}\int_{-\infty}^{\infty}(\tilde{\mu}-x)^2 e^{-(x+\alpha(\tilde{\mu}-\mu)-\tilde{\mu})^2/2}\mathrm{d}x \\ &= \frac{1}{\sqrt{2\pi}}\,e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}\int_{-\infty}^{\infty}(\alpha(\tilde{\mu}-\mu)-t)^2 e^{-t^2/2}\mathrm{d}t \\ &= \frac{1}{\sqrt{2\pi}}\,e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}\,2\int_0^{\infty}(\alpha^2(\tilde{\mu}-\mu)^2+t^2)\,e^{-t^2/2}\mathrm{d}t \\ &= \frac{1}{\sqrt{2\pi}}\,e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}\,2\int_0^{\infty}(\alpha^2(\tilde{\mu}-\mu)^2+2z)\frac{1}{\sqrt{2z}}\,e^{-z}\mathrm{d}z \\ &= \left[1+\alpha^2(\tilde{\mu}-\mu)^2\right]e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}. \end{aligned} \tag{50}$$

Using (48) and (50) we obtain the influence function

$$\mathrm{IF}(x;\tilde{T}_{\alpha,\mu},P_{\tilde{\mu}}) = \frac{-\,\psi_{\alpha,\mu}(x,\tilde{\mu})}{\boldsymbol{I}_{\alpha,\mu}(P_{\tilde{\mu}})} = \frac{(x-\tilde{\mu})e^{\alpha(\tilde{\mu}-\mu)(\tilde{\mu}+\mu-2x)/2}+\alpha(\tilde{\mu}-\mu)e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}}{(1+\alpha^2(\tilde{\mu}-\mu)^2)e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}}$$

and the asymptotic variance

$$
\begin{aligned}
\mathrm{V}(\tilde{T}_{\alpha,\mu}, P_{\tilde{\mu}}) &= \boldsymbol{I}_{\alpha,\mu}^{-2}(P_{\tilde{\mu}}) \int \psi_{\alpha,\mu}^2(x,\tilde{\mu})\,\mathrm{d}P_{\tilde{\mu}} \\
&= \frac{\int_{-\infty}^{\infty} [(\tilde{\mu}-x)e^{\alpha(\tilde{\mu}-\mu)(\tilde{\mu}+\mu-2x)/2} - \alpha(\tilde{\mu}-\mu)e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2}]^2\, e^{-(x-\tilde{\mu})^2/2}\,\mathrm{d}x}{\sqrt{2\pi}\,\boldsymbol{I}_{\alpha,\mu}^2(P_{\tilde{\mu}})} \\
&= \frac{1}{\sqrt{2\pi}\,\boldsymbol{I}_{\alpha,\mu}^2(P_{\tilde{\mu}})} \int_{-\infty}^{\infty} (\tilde{\mu}-x)^2 e^{\alpha(\tilde{\mu}-\mu)(\tilde{\mu}+\mu-2x)-(x-\tilde{\mu})^2/2}\,\mathrm{d}x \\
&\quad - \frac{\sqrt{2}}{\sqrt{\pi}\,\boldsymbol{I}_{\alpha,\mu}^2(P_{\tilde{\mu}})}\,\alpha(\tilde{\mu}-\mu) \int_{-\infty}^{\infty} (\tilde{\mu}-x) e^{(\alpha(\tilde{\mu}-\mu)(2\mu-2x+\alpha(\tilde{\mu}-\mu))-(x-\tilde{\mu})^2)/2}\,\mathrm{d}x \\
&\quad + \frac{1}{\boldsymbol{I}_{\alpha,\mu}^2(P_{\tilde{\mu}})}\,\alpha^2(\tilde{\mu}-\mu)^2 e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2} \\
&= \frac{1}{\boldsymbol{I}_{\alpha,\mu}^2(P_{\tilde{\mu}})} \left[ e^{\alpha(2\alpha-1)(\tilde{\mu}-\mu)^2}(4\alpha^2(\tilde{\mu}-\mu)^2+1) - \alpha^2(\tilde{\mu}-\mu)^2 e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2} \right] \\
&= \frac{1}{[1+\alpha^2(\tilde{\mu}-\mu)^2]^2} \left[ e^{\alpha^2(\tilde{\mu}-\mu)^2}(4\alpha^2(\tilde{\mu}-\mu)^2+1) - \alpha^2(\tilde{\mu}-\mu)^2 \right]. \qquad (51)
\end{aligned}
$$

**Example 1** For the parent distribution $P_0$ of the normal family we obtain by plugging $\tilde{\mu} = 0$ in formulas (50), (**??**) and (51) the following:

$$\boldsymbol{I}_{\alpha,\mu}(P_0) = \left[1 + \alpha^2\mu^2\right] e^{\alpha(\alpha-1)\mu^2/2},$$

$$\mathrm{IF}(x; \tilde{T}_{\alpha,\mu}, P_0) = \frac{x\, e^{-\alpha\mu(\mu-2x)/2} - \alpha\,\mu\, e^{\alpha(\alpha-1)\mu^2/2}}{(1+\alpha^2\mu^2)\, e^{\alpha(\alpha-1)\mu^2/2}},$$

$$\mathrm{V}(\tilde{T}_{\alpha,\mu}, P_0) = \frac{1}{(1+\alpha^2\mu^2)^2} \left[ e^{\alpha^2\mu^2}(4\alpha^2\mu^2 + 1) - \alpha^2\mu^2 \right].$$

**Example 2** Lets now consider a mixture $Q = (1-\varepsilon)P_0 + \varepsilon P_{0,\sigma}$, $0 \le \varepsilon \le 1/2$, $\sigma \ge 1$, with density

$$q(x) = (1-\varepsilon)\,\frac{e^{-x^2/2}}{\sqrt{2\pi}} + \varepsilon\,\frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}.$$

To derive the influence function and asymptotic variance of estimator $\tilde{T}_{\alpha,\mu}$ $(0 < \alpha < 1)$ we shall use the basic formulas (31), (32) and (35).

Firstly, we will find $\tilde{T}_{\alpha,\mu}(Q)$ as a solution of equation

$$\int \psi_{\alpha,\mu}(x, \tilde{\mu})\, \mathrm{d}Q(x) = 0$$

where, by using (48),

$$\begin{aligned}
&\textstyle\int \psi_{\alpha,\mu}(x,\tilde{\mu})\,\mathrm{d}Q(x) = \\
&= \int_{-\infty}^{\infty} \left( (\tilde{\mu} - x)\, e^{\alpha(\tilde{\mu}-\mu)(\tilde{\mu}+\mu-2x)/2} - \alpha(\tilde{\mu}-\mu)\, e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2} \right) q(x)\,\mathrm{d}x \\
&= -\alpha(\tilde{\mu}-\mu)\, e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2} + \int_{-\infty}^{\infty} (\tilde{\mu}-x)\, e^{\alpha(\tilde{\mu}-\mu)(\tilde{\mu}+\mu-2x)/2} \left( \frac{(1-\varepsilon)e^{-x^2/2}}{\sqrt{2\pi}} + \frac{\varepsilon e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \right) \mathrm{d}x \\
&= -\alpha(\tilde{\mu}-\mu)\, e^{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2} + (1-\varepsilon)(\tilde{\mu} + \alpha(\tilde{\mu}-\mu))\, e^{(\alpha^2(\tilde{\mu}-\mu)^2 + \alpha(\tilde{\mu}^2-\mu^2))/2} \\
&\quad + \varepsilon(\tilde{\mu} + \alpha\sigma^2(\tilde{\mu}-\mu))\, e^{(\alpha^2\sigma^2(\tilde{\mu}-\mu)^2 + \alpha(\tilde{\mu}^2-\mu^2))/2}.
\end{aligned} \tag{52}$$

Using this formula, we are able to solve the equation only for $\alpha = 0$ which gives us the solution $\tilde{\mu} = 0$, or for escort parameter $\mu = 0$, which simplifies the expression (52) and we obtain the equation

$$\begin{aligned}
-\alpha\tilde{\mu}\, e^{\alpha(\alpha-1)\tilde{\mu}^2/2} + (1-\varepsilon)(\tilde{\mu} + \alpha\tilde{\mu})\, e^{(\alpha^2\tilde{\mu}^2 + \alpha\tilde{\mu}^2)/2} + \varepsilon(\tilde{\mu} + \alpha\sigma^2\tilde{\mu})\, e^{(\alpha^2\sigma^2\tilde{\mu}^2 + \alpha\tilde{\mu}^2)/2} &= 0 \\
\tilde{\mu}\left[ e^{\alpha^2\tilde{\mu}^2/2} \left( \underbrace{\alpha(e^{\alpha\tilde{\mu}^2/2} - e^{-\alpha\tilde{\mu}^2/2})}_{\ge 0} + \underbrace{e^{\alpha\tilde{\mu}^2/2}(1 - \varepsilon(1+\alpha))}_{> 0} \right) + \underbrace{\varepsilon(1+\alpha\sigma^2)\, e^{(\alpha^2\sigma^2\tilde{\mu}^2 + \alpha\tilde{\mu}^2)/2}}_{\ge 0} \right] &= 0
\end{aligned}$$

and receive the unique solution $\tilde{\mu} = 0$. In the sequel, we will consider only $\alpha = 0$ or $\mu = 0$, so that $\tilde{T}_{\alpha,\mu}(Q) = 0$.

Applying this result to (31), we receive

$$
\begin{aligned}
\boldsymbol{I}_{\alpha,\mu}(Q) &= \int \mathring{\boldsymbol{\psi}}_{\alpha,\mu}(x, \tilde{T}_{\alpha,\mu}(Q))\,\mathrm{d}Q = \int \mathring{\boldsymbol{\psi}}_{\alpha,\mu}(x,0)\,\mathrm{d}Q \\
&= \int_{-\infty}^{\infty} \Big((1+\alpha x^2)\, e^{-\alpha\mu(\mu-2x)/2} - \alpha(1+\alpha(\alpha-1)\mu^2)\, e^{\alpha(\alpha-1)\mu^2/2}\Big)\, q(x)\,\mathrm{d}x \\
&= (1-\varepsilon)\boldsymbol{I}_{\alpha,\mu}(P_0) \\
&\quad + \varepsilon \int_{-\infty}^{\infty} \Big((1+\alpha x^2)\, e^{-\alpha\mu(\mu-2x)/2} - \alpha(1+\alpha(\alpha-1)\mu^2)\, e^{\alpha(\alpha-1)\mu^2/2}\Big) \left(\frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\right) \mathrm{d}x \\
&= (1-\varepsilon)\boldsymbol{I}_{\alpha,\mu}(P_0) - \varepsilon\alpha(1+\alpha(\alpha-1)\mu^2)\, e^{\alpha(\alpha-1)\mu^2/2} \\
&\quad + \varepsilon \int_{-\infty}^{\infty} (1+\alpha x^2)\, e^{-\alpha\mu(\mu-2x)/2} \left(\frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\right) \mathrm{d}x \\
&= e^{\alpha(\alpha-1)\mu^2/2} \left[(1-\varepsilon)(1+\alpha^2\mu^2) - \varepsilon\alpha(1+\alpha(\alpha-1)\mu^2)\right] \\
&\quad + \varepsilon e^{\alpha\mu^2(\alpha\sigma^2-1)/2}(1+\alpha^3\sigma^4\mu^2+\alpha\sigma^2)\,.
\end{aligned}
$$

Consequently we have influence function

$$
\mathrm{IF}(x; \tilde{T}_{\alpha,\mu}, Q) = -\boldsymbol{I}^{-1}_{\alpha,\mu}(Q)\, \boldsymbol{\psi}_{\alpha,\mu}(x,0) = \frac{x\, e^{-\alpha\mu(\mu-2x)/2} - \alpha\,\mu\, e^{\alpha(\alpha-1)\mu^2/2}}{\boldsymbol{I}_{\alpha,\mu}(Q)}
$$

and the asymptotic variance

$$
\begin{aligned}
\mathrm{V}(\tilde{T}_{\alpha,\mu}, Q) &= \boldsymbol{I}^{-2}_{\alpha,\mu}(Q) \int \boldsymbol{\psi}^2_{\alpha,\mu}(x,0)\,\mathrm{d}Q \\
&= \boldsymbol{I}^{-2}_{\alpha,\mu}(Q) \int_{-\infty}^{\infty} \Big(-x\, e^{-\alpha\mu(\mu-2x)/2} + \alpha\,\mu\, e^{\alpha(\alpha-1)\mu^2/2}\Big)^2 q(x)\,\mathrm{d}x \\
&= (1-\varepsilon)\; \boldsymbol{I}^{-2}_{\alpha,\mu}(Q)\; \boldsymbol{I}^{2}_{\alpha,\mu}(P_0)\; \mathrm{V}(\tilde{T}_{\alpha,\mu}, P_0) \\
&\quad + \varepsilon\, \boldsymbol{I}^{-2}_{\alpha,\mu}(Q) \int_{-\infty}^{\infty} \Big(-x\, e^{-\alpha\mu(\mu-2x)/2} + \alpha\,\mu\, e^{\alpha(\alpha-1)\mu^2/2}\Big)^2 \left(\frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\right) \mathrm{d}x \\
&= (1-\varepsilon)\; \boldsymbol{I}^{-2}_{\alpha,\mu}(Q) \Big[\, e^{\alpha(2\alpha-1)\mu^2}(4\alpha^2\mu^2+1) - \alpha^2\mu^2 e^{\alpha(\alpha-1)\mu^2}\Big] \\
&\quad + \varepsilon\; \boldsymbol{I}^{-2}_{\alpha,\mu}(Q) \Big[\alpha^2\mu^2 e^{\alpha(\alpha-1)\mu^2} + \sigma^2(4\alpha^2\mu^2\sigma^2+1)e^{\alpha(2\alpha\sigma^2-1)\mu^2} - 2\alpha^2\mu^2\sigma^2 e^{\alpha(\alpha\sigma^2+\alpha-2)\mu^2/2}\Big]\,.
\end{aligned}
$$

# 4 COMPARATIVE SIMULATION STUDY

This chapter presents the results obtained by applying the methods of Broniatowski & Keziou and Liese & Vajda introduced in the previous chapters. We orient the study at the power subdivergence and power superdivergence estimators of location given by

$$\mu_{0,n} = \tilde{\mu}_{0,\mu,n} = \bar{\boldsymbol{X}}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

and

$$\mu_{\alpha,n} = \operatorname{argmax}_{\mu}\operatorname{inf}_{\tilde{\mu}}\, M_{\alpha,\mu}(P_n,\tilde{\mu}) \qquad \tilde{\mu}_{\alpha,\mu,n} = \operatorname{argmin}_{\tilde{\mu}}\, M_{\alpha,\mu}(P_n,\tilde{\mu})$$

for $\alpha > 0$ where

$$M_{\alpha,\mu}(P_n,\tilde{\mu}) = \frac{1}{1-\alpha}\left(\exp\left\{\alpha(\alpha-1)(\tilde{\mu}-\mu)^2/2\right\}\right) + \frac{1}{n\alpha}\sum_{i=1}^{n}\exp\left\{\alpha(\tilde{\mu}-\mu)(\tilde{\mu}+\mu-2X_i)/2\right\}$$

and $X_1, ..., X_n$ are observations generated by the convex mixtures

$$P_\varepsilon = (1-\varepsilon)P + \varepsilon Q.$$

Here $P$ is a standard normal model with location $\mu = 0$ and scale $\sigma = 1$, further denoted by $N(0,1)$, and Q is successively normal ($N(0,9), N(0,100)$), logistic ($Lo(0,1)$), and Cauchy ($C(0,1)$) distribution. We use the contamination levels of 0, 1, 5, 10, 20, 30 percent , i.e. $\varepsilon$ takes on the values 0, 0.01, 0.05, 0.1, 0.2, and 0.3. The selected sample sizes $n$ are 20, 50, 100, 200, 500.

For min$\bar{\mathrm{D}}_\alpha$-estimators $\mu_{\alpha,n}$ we take into account only power parameters 0, 0.01, 0.05, 0.1, 0.2, and 0.5. In the case of max$\underline{\mathrm{D}}_\alpha$-estimators $\tilde{\mu}_{\alpha,\mu,n}$ we consider the same values of power parameter and in addition to that we select the escort parameters $\mu$ = 0, 0.1, 0.2, 0.5, 1 and finally $\mu = \bar{\boldsymbol{X}}_n$ (MLE).

To evaluate the behavior of power superdivergence (or power subdivergence) estimators we generate K different data samples (K=100 or K=1000) to gain K different estimates (further indexed by $^{(k)}$) and we compute the mean

$$m(\mu) = \frac{1}{K}\sum_{k=1}^{K}\mu_{\alpha,n}^{(k)}$$

and standard deviation

$$s(\mu) = \sqrt{\frac{1}{K}\sum_{k=1}^{K}(\mu_{\alpha,n}^{(k)} - m(\mu))^2}$$

of the min$\bar{\text{D}}_\alpha$-estimators (or max$\underline{\text{D}}_\alpha$-estimators) and maximum likelihood estimators $\bar{\boldsymbol{X}}_n^{(k)}$. Using these we receive the relative empirical efficiency

$$\text{eref}(\mu) = \frac{\frac{1}{K}\sum_{k=1}^K(\bar{\boldsymbol{X}}_n^{(k)})^2}{\frac{1}{K}\sum_{k=1}^K(\mu_{\alpha,n}^{(k)})^2} = \frac{s^2(\bar{\boldsymbol{X}}_n) + m^2(\bar{\boldsymbol{X}}_n)}{s^2(\mu) + m^2(\mu)}.$$

## 4.1 Software solution and algorithms

In this section we shall describe in detail the primary methods and procedures making up the substantial part of the program. For the implementation of the software solution we used the C++ language and the C++ Builder 3.1 of the Borland International company. The program was created and debugged at the Windows 2000 platform.

### 4.1.1 Basic algorithms

In the following paragraphs we give the core algorithms for the maximum subdivergence estimators and minimum superdivergence estimators.

*Power subdivergence estimator algorithm:*

1) We choose the contaminated and contaminating distibutions and set their parameters (in our case $(N(0,1), N(0,9), N(0,100), Lo(0,1), C(0,1),))$, then we enter the value of the contamination parameter $\varepsilon$ and the value of the escorting parameter $\mu$. In addition to this, we determine the number of estimates $K$.

2) For $n$ equal to 20, 50, 100, 200, 500 successively and power parameters $\alpha = 0$, 0.01, 0.05, 0.1, 0.2, and 0.5, we repeat the following steps a) - c) K-times to receive K different estimates for each value of $n$ and each value of $\alpha$.

a) We generate the pseudorandom data sample $X_1, ..., X_n$ of the observations on the mixture

$$P_\varepsilon = (1-\varepsilon)P + \varepsilon Q.$$

b) From the data sample we compute the maximum likelihood estimate $\bar{\boldsymbol{X}}_n$

c) We pass the sample and the escort parameter to the optimization method minimizing over $\tilde{\mu}$ (using a separate function to obtain the values of $M_{\alpha,\mu}(P_n, \tilde{\mu})$) and generating the estimate

$$\tilde{\mu}_{\alpha,\mu,n} = \text{argmin}_{\tilde{\mu}}\ M_{\alpha,\mu}(P_n, \tilde{\mu})$$

for $\alpha > 0$ or the maximum likelihood estimate for $\alpha = 0$.

3) We compute

$$m(\tilde{\mu}) = \frac{1}{K}\sum_{k=1}^K \tilde{\mu}_{\alpha,\mu,n}^{(k)}\ , \qquad s(\tilde{\mu}) = \sqrt{\frac{1}{K}\sum_{k=1}^K(\tilde{\mu}_{\alpha,\mu,n}^{(k)} - m(\tilde{\mu}))^2}\ ,$$

$$m(\bar{\boldsymbol{X}}_n) = \frac{1}{K}\sum_{k=1}^{K} \bar{\boldsymbol{X}}_n^{(k)} , \qquad s(\bar{\boldsymbol{X}}_n) = \sqrt{\frac{1}{K}\sum_{k=1}^{K} (\bar{\boldsymbol{X}}_n^{(k)} - m(\bar{\boldsymbol{X}}_n))^2} ,$$

and

$$\mathrm{eref}(\tilde{\mu}) = \frac{s^2(\bar{\boldsymbol{X}}_n) + m^2(\bar{\boldsymbol{X}}_n)}{s^2(\tilde{\mu}) + m^2(\tilde{\mu})}$$

for every $n$ and $\alpha$, and tabulate the values of $m(\tilde{\mu})$, $s(\tilde{\mu})$, and $\mathrm{eref}(\tilde{\mu})$.

*Power superdivergence estimator algorithm:*

1) We choose the contaminated and contaminating distibutions and set their parameters (in our case $(N(0,1), N(0,9), N(0,100), Lo(0,1), C(0,1),))$, then we enter the value of the contamination parameter $\varepsilon$ and the number of estimates $K$.

2) For $n$ equal to 20, 50, 100, 200, 500 successively and power parameters $\alpha = 0$, 0.01, 0.05, 0.1, 0.2, and 0.5, we repeat the following steps a) - c) K-times to receive K different estimates for each value of $n$ and each value of $\alpha$.

a) We generate the pseudorandom data sample $X_1, ..., X_n$ of the observations on the mixture

$$P_\varepsilon = (1-\varepsilon)P + \varepsilon Q.$$

b) From the data sample we compute the maximum likelihood estimate $\bar{\boldsymbol{X}}_n$

c) We pass the sample to the optimization method minimalizing over $\mu$ the function

$$-1 \cdot \inf_{\tilde{\mu}} M_{\alpha,\mu}(P_n, \tilde{\mu}).$$

Evaluation of this function requires another call of the optimization method minimalizing over $\tilde{\mu}$ the function $M_{\alpha,\mu}(P_n, \tilde{\mu})$. By this procedure we receive the estimate

$$\mu_{\alpha,n} = \mathrm{argmax}_\mu \mathrm{inf}_{\tilde{\mu}} \, M_{\alpha,\mu}(P_n, \tilde{\mu})$$

for $\alpha > 0$. For $\alpha = 0$ we get the maximum likelihood estimate again.

3) We compute

$$m(\mu) = \frac{1}{K}\sum_{k=1}^{K} \mu_{\alpha,n}^{(k)} , \qquad s(\mu) = \sqrt{\frac{1}{K}\sum_{k=1}^{K} (\mu_{\alpha,n}^{(k)} - m(\mu))^2} ,$$

$$m(\bar{\boldsymbol{X}}_n) = \frac{1}{K}\sum_{k=1}^{K} \bar{\boldsymbol{X}}_n^{(k)} , \qquad s(\bar{\boldsymbol{X}}_n) = \sqrt{\frac{1}{K}\sum_{k=1}^{K} (\bar{\boldsymbol{X}}_n^{(k)} - m(\bar{\boldsymbol{X}}_n))^2} ,$$

and

$$\mathrm{eref}(\mu) = \frac{s^2(\bar{\boldsymbol{X}}_n) + m^2(\bar{\boldsymbol{X}}_n)}{s^2(\mu) + m^2(\mu)}$$

for every $n$ and $\alpha$, and tabulate the values of $m(\mu)$, $s(\mu)$, and $\mathrm{eref}(\mu)$.

The formulas for statistical models used, the procedures for pseudorandom number generation and function minimization will be described more closely in the next sections.

### 4.1.2 Families of probability distributions

In the sequel, we give the formulas for the normal, logistic and Cauchy models.

**Normal distribution** - $N(\mu, \sigma^2), \quad \mu, \sigma \in \mathbb{R}, \quad \sigma^2 > 0$

$$f_{\mu,\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$F_{\mu,\sigma^2}(x) = \int_{-\infty}^{x} f_{\mu,\sigma^2}(t)\, dt$$

**Logistic distribution** - $O(\alpha, \beta), \quad \alpha, \beta \in \mathbb{R}, \quad \beta > 0$

$$f_{\alpha,\beta}(x) = \frac{1}{\beta} \frac{e^{-\frac{x-\alpha}{\beta}}}{\left(1+e^{-\frac{x-\alpha}{\beta}}\right)^2}$$

$$F_{\alpha,\beta}(x) = \left(1 + e^{-\frac{x-\alpha}{\beta}}\right)^{-1}, \qquad F^{-1}_{\alpha,\beta}(x) = \beta\, \ln(\tfrac{x}{1-x}) + \alpha$$

**Cauchy distribution** - $C(u, v), \quad u, v \in \mathbb{R}, \quad v > 0$

$$f_{u,v}(x) = \frac{1}{\pi v} \frac{1}{1+\left(\frac{x-u}{v}\right)^2}$$

$$F_{u,v}(x) = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{x-u}{v}\right), \qquad F^{-1}_{u,v}(x) = v\, \tan[\, \pi\, (x - 1/2)\,] + u$$

### 4.1.3 Generation of pseudorandom numbers

In the following paragraphs we describe the methods how to obtain individual realization of a random variable $X$ governed by one of the considered probability distributions, and also the way to receive the sample of observations on the mixture $P_\varepsilon = (1-\varepsilon)P + \varepsilon Q$.

The essential part is the generation of independent realizations of $X \sim U(0,1)$ for which we use the well known pseudorandom number generator of Wichmann & Hill ([10]) with the cycle of $7 \cdot 10^{12}$ :

$$U_{i+1} = \left(\frac{X_{i+1}}{30269} + \frac{Y_{i+1}}{30307} + \frac{Z_{i+1}}{30323}\right) \bmod 1$$

$$X_{i+1} = (\, 171\, X_i\,) \bmod 30269$$
$$Y_{i+1} = (\, 172\, Y_i\,) \bmod 30307$$
$$Z_{i+1} = (\, 170\, Z_i\,) \bmod 30323$$

This generator has been widely used in the last years and implemented in many statistical applications. Wichmann & Hill generator has been tested by the Diehard test for random generators and passed successfully ([7]). Its cycle length is now considered inadequate for some purposes and the authors work to improve it, but for our purposes it is sufficient and we consider the use of this generator in this situation as adequate.

To obtain $X \sim Lo(a,b)$ or $X \sim C(a,b)$, $a,b \in \mathbb{R}$, $b > 0$ we apply the following theorem ([9]).

**Theorem 3.** *Let $X$ be a random variable with the absolutely continuous distribution $F$. Then the random variable $Y = F(X)$ is uniformly distributed in (0,1).*

In consequence we use this simple algorithm to get $X \sim Lo(a,b)$, resp. $X \sim C(a,b)$:
1) We generate one realization $y$ of $Y \sim U(0,1)$.
2) We find $x = F^{-1}(y)$, where $F^{-1}$ is the inverse function to the distribution function of logistic, resp. Cauchy distribution. This $x$ is the desired realization of $X \sim Lo(a,b)$, resp. $X \sim C(a,b)$.

This is obviously not possible for distributions where the inverse function $F^{-1}$ is not available, such as the normal distribution. According to [9], it is possible to obtain two independent realizations $X_1, X_2 \sim N(0,1)$ by formulas

$$X_1 = \sqrt{-2\ln Y_1}\ \sin(2\pi Y_2) \quad \text{and} \quad X_2 = \sqrt{-2\ln Y_1}\ \cos(2\pi Y_2)$$

where $Y_1, Y_2$ are independent realizations of $U(0,1)$.

To get the realization of normal distribution with location $\mu$ and scale $\sigma$, we use the relation

$$\mu + \sigma\, X \sim N(\mu, \sigma^2)\ , \quad \text{for} \quad X \sim N(0,1).$$

An observation on the mixture $P_\varepsilon = (1-\varepsilon)P + \varepsilon Q$ is received in two steps:
1) We generate one realization $y$ of $Y \sim U(0,1)$
2) If $y \leq 1-\varepsilon$, we generate a realization $x$ of $X \sim P$.
If $y > 1-\varepsilon$, we generate a realization $x$ of $X \sim Q$.

By repeating $n$-times we gain the whole sample of observations on the mixture considered.

#### 4.1.4 Minimization method

Lets now consider a function $f : \mathbb{R} \to \mathbb{R}$ and lets describe the method how to find

$$x_{min} = \mathrm{argmin}_{x\in\mathbb{R}}\ f(x).$$

Already in [4] we were looking for a good optimization method with a reasonable trade-off between efficiency and accurateness. The best solution turned out to be a combination of the successively thickening grids and the gradient method. First, we describe the general algorithm, and then we specify the constants and other details of our case.

*General algorithm:*

1) We spread out the grid with the mesh length $\varepsilon$ on a restricted area $\langle a,b\rangle \subset \mathbb{R}$ and compute the values of the minimized function $f$. (The interval $\langle a,b\rangle$ must be chosen very carefully to be wide enough to ensure that we find the minimum $x_{min}$, but not too wide to slow the procedure down considerably.)

2) A certain number $p > 1$ of points with the lowest function value is stored in a sorted stack $\{x_1^{(0)}, ..., x_p^{(0)}\} \subset \langle a, b \rangle$.

3) For each of the initial points $x_i^{(0)}$ we use the simplified gradient method to reach the local minimum:

a) We set the initial value of the step length $\varepsilon_0$ and we repeat the steps i) - ii)

i) For $j = 0, 1, 2, ...$ we iterate the following step

- Compute the values $f(x_i^{(j)} - \varepsilon_k)$ and $f(x_i^{(j)} + \varepsilon_k)$ and move to the point where the value is the lowest, i.e. $x_i^{(j+1)} = \text{argmin}\{f(x_i^{(j)} - \varepsilon_k), f(x_i^{(j)}), f(x_i^{(j)} + \varepsilon_k)\}$.

until we get into the point for which the function value is lower then the function values of the surrounding points, i.e. $x_i^{(j+1)} = x_i^{(j)}$.

ii) We shorten the step $\varepsilon_{k+1} = const \cdot \varepsilon_k$ $(0 < const < 1)$.

until we locate the minimum with the prescribed accuracy, i.e. $\varepsilon_k < 10^{-n}$, $n > 1$.

4) Now we have $p$ local minima from which we gain the global minimum $x_{min}$ of the function (at the limited parameter space).

This method is very simple in general, but its implementation carries along several unpleasant complications. It's necessary to avoid all the possibilities of never-ending cycles (hence the restrictions to the parameter space, accuracy, etc.) or the possibilities of overflow in case of large values (we used the largest data type with values from $3.4 \cdot 10^{-4932}$ to $1.1 \cdot 10^{4932}$ but even that was not enough in some cases).

We also tested the constants (mesh length, step length, etc.) to make them reasonably rough to proceed as fast as possible and still receive the same results as with the fine grid and little steps. The final setting was chosen as follows : mesh legth $\varepsilon = 0.05$, number of minimal points $p = 5$, the initial step length $\varepsilon_0 = 0.05$, accuracy of the local minima $10^{-4}$, and $const = 0.5$ (at first, we used $const = 0.8$, but the progress was too slow and computation took too much time, therefore, we tested a factor 0.5 and received the same results as in the previous case, hence, we use $const = 0.5$).

The last problem that needs to be clarified is the method of restricting $\mathbb{R}$ to $\langle a, b \rangle$. Since we work with symmetric distribution N(0,1) and we contaminate the data with symmetric distributions N(0,9), N(0,100), Lo(0,1), and C(0,1), it is reasonable to expect the estimate to be "somewhere around" 0, but we need to inspect the minimalized function $M_{\alpha,\mu}(P_n, \tilde{\mu})$ nevertheless. Figure 1 shows the behavior of this function and location of $\tilde{\mu}_{\alpha,\mu,n} = \text{argmin}_{\tilde{\mu}}\ M_{\alpha,\mu}(P_n, \tilde{\mu})$ for several models and parameter settings :

1) 0.95 N(0,1) + 0.05 N(0,9), escort parameter $m = 0.2$, power parameter $\alpha = 0.01$
data ($n = 20$):
-1.858, -1.395, -1.134, -0.920, -0.706, -0.682, -0.564, -0.389, -0.121, -0.109, -0.050, -0.023, 0.098, 0.150, 0.236, 0.259, 1.192, 3.951, 4.071, 4.950

2) 0.9 N(0,1) + 0.1 N(0,100), escort parameter $m = 0.1$, power parameter $\alpha = 0.01$
data ($n = 50$):
-22.853, -7.226, -5.258, -2.147, -1.858, -1.711, -1.671, -1.597, -1.523, -1.395, -1.367, -1.330, -1.134, -1.037, -0.931, -0.927, -0.920, -0.706, -0.682, -0.639, -0.615, -0.603, -0.564, -0.389, -0.349, -0.121, -0.109, -0.050, -0.023, 0.009, 0.090, 0.098, 0.100, 0.150, 0.236, 0.259, 0.352, 0.363, 0.480, 0.898, 1.042, 1.058, 1.128, 1.136, 1.192, 1.505, 2.441, 13.170, 13.570, 16.499

3) 0.8 N(0,1) + 0.2 Lo(0,1), escort parameter $m = 0.5$, power parameter $\alpha = 0.05$
data ($n = 50$):
-3.484, -2.727, -2.676, -2.037, -1.998, -1.586, -1.330, -1.265, -1.161, -1.051, -1.044, -1.037, -1.007, -0.995, -0.980, -0.927, -0.696, -0.669, -0.630, -0.624, -0.594, -0.492, -0.428, -0.419, -0.402, -0.363, -0.337, -0.291, -0.259, -0.133, -0.109, 0.048, 0.265, 0.307, 0.350, 0.437, 0.512, 0.600, 0.676, 0.687, 0.904, 1.042, 1.097, 1.192, 1.428, 1.505, 1.700, 2.331, 2.495, 4.927

4) 0.99 N(0,1) + 0.01 C(0,1), escort parameter $m = 1$, power parameter $\alpha = 0.2$
data ($n = 100$):
-2.902, -2.289, -2.015, -1.998, -1.858, -1.830, -1.694, -1.682, -1.624, -1.599, -1.586, -1.495, -1.395, -1.264, -1.252, -1.134, -1.121, -1.073, -1.070, -1.044, -1.011, -0.979, -0.979, -0.920, -0.822, -0.816, -0.789, -0.775, -0.706, -0.682, -0.665, -0.610, -0.564, -0.526, -0.476, -0.452, -0.433, -0.419, -0.392, -0.389, -0.388, -0.367, -0.360, -0.359, -0.344, -0.322, -0.312, -0.304, -0.292, -0.158, -0.157, -0.135, -0.133, -0.121, -0.109, -0.050, -0.026, -0.023, 0.030, 0.053, 0.098, 0.150, 0.176, 0.195, 0.213, 0.221, 0.236, 0.254, 0.259, 0.339, 0.406, 0.427, 0.440, 0.473, 0.477, 0.514, 0.600, 0.644, 0.695, 0.863, 0.890, 0.919, 0.999, 1.078, 1.192, 1.249, 1.275, 1.312, 1.317, 1.369, 1.397, 1.399, 1.428, 1.436, 1.458, 1.534, 1.650, 1.747, 1.947, 3.234

As we can see from Figure 1, for the purpose of our simulations it is sufficient to search for the minimum inside the interval $\langle -5, 5\rangle$.

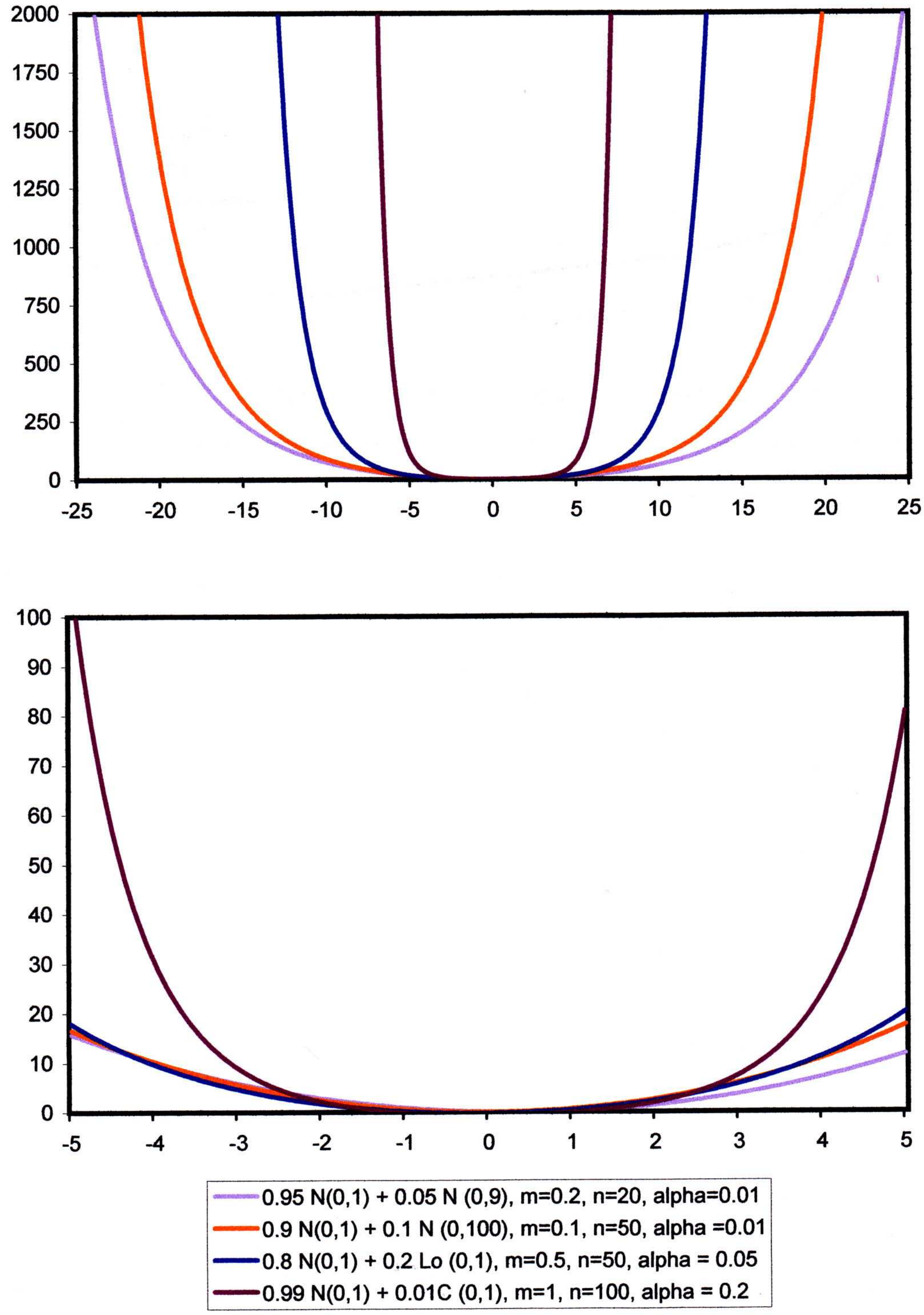


Figure 1: Function $M_{\alpha,\mu}(P_n, \tilde{\mu})$ under different data sets and parameter settings

## 4.2 Tabulated results for power subdivergence estimators

In this section we state the conclusions based on the tables computed for power subdivergence estimators.

Tables 1 - 30 show the development of consistency, efficiency and robustness in the case of mixture $(1-\varepsilon)N(0,1)+\varepsilon N(0,9)$ for moving value of escorting parameter $\mu = 0$, 0.1, 0.2, 0.5, 1 and the contamination parameter $\varepsilon = 0$, 0.01, 0.05, 0.1, 0.2, and 0.3.

We see that for power parameter $\alpha = 0$ the estimates coincide with MLE, i.e. $\text{eref}(\tilde{\mu}) = 1$, as was expected. In case of escort parameter $\mu = 0$, the $\max \underline{D}_\alpha$-estimators for the uncontaminated data still more or less copy the behavior of MLE even for values of $\alpha > 0$, but as the contamination grows, we see that the mean and standard deviation of $\max \underline{D}_\alpha$-estimator move apart from MLE taking on lower values than maximum likelihood estimate of the contaminated data. In case of $m(\tilde{\mu})$ the difference is only slight (yet favourable, cf. Table 26), but in case of $s(\tilde{\mu})$ is the difference apparent (cf. Figure 2) and causes a fair increase in empirical relative efficiency.

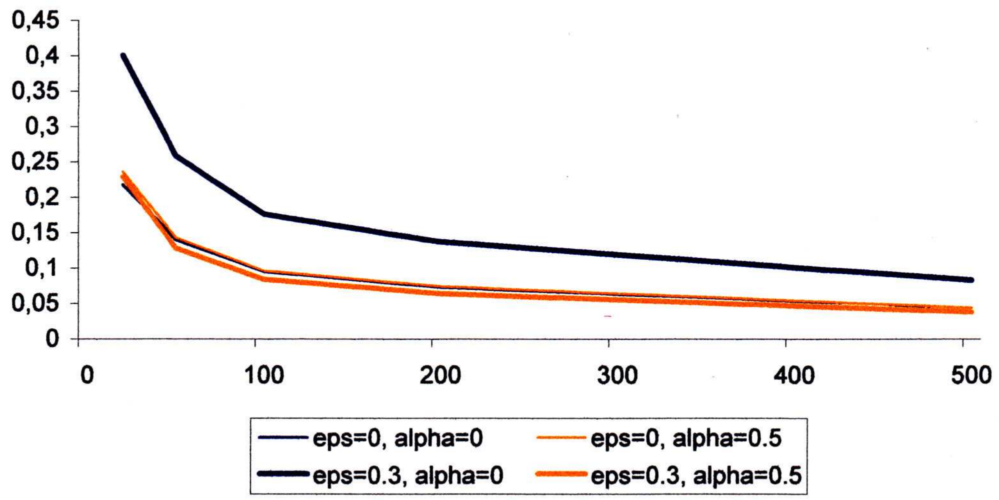


Figure 2: Dependency of standard deviation of the $\max \underline{D}_\alpha$-estimators with escort parameter $\mu = 0$ on sample size $n$ for data distributed by $(1-\varepsilon)N(0,1)+\varepsilon N(0,9)$

Figure 3 displays the development of eref($\tilde{\mu}$) for different values of power parameter $\alpha$ showing us that the robustness tendency is growing stronger with $\alpha$ increasing. Since the dependence on sample size $n$ is almost constant for $n > 50$, we present in Figure 4 the value of eref($\tilde{\mu}$) only for $n = 500$ as a function of contamination parameter $\varepsilon$ for different levels of $\alpha$ This shows the rising efficiency of max$\underline{\mathrm{D}}_\alpha$-estimator (compared to MLE with $\alpha = 0$) with increasing contamination.

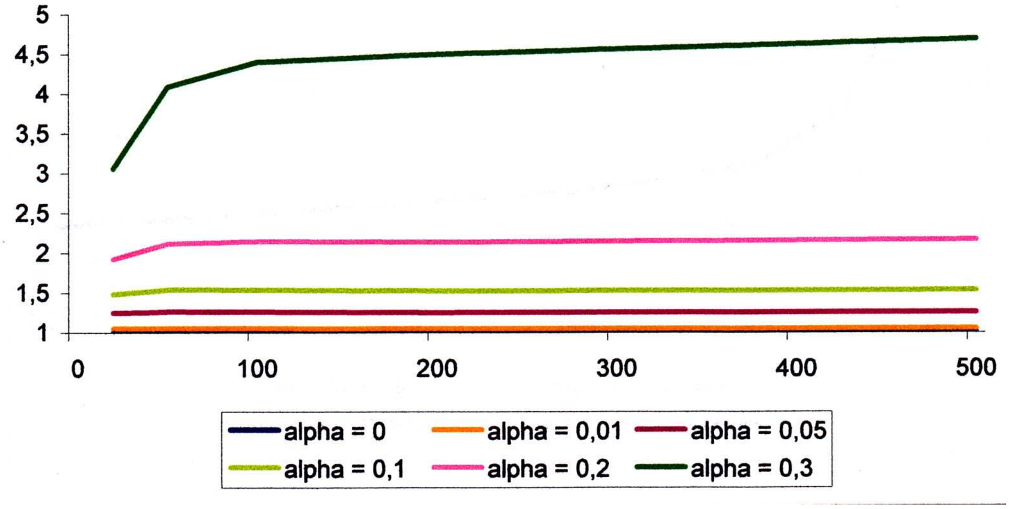


Figure 3: Dependency of empirical relative efficiency of the max$\underline{\mathrm{D}}_\alpha$-estimators with escort parameter $\mu = 0$ on sample size $n$ for data distributed by $0.7N(0,1) + 0.3N(0,9)$

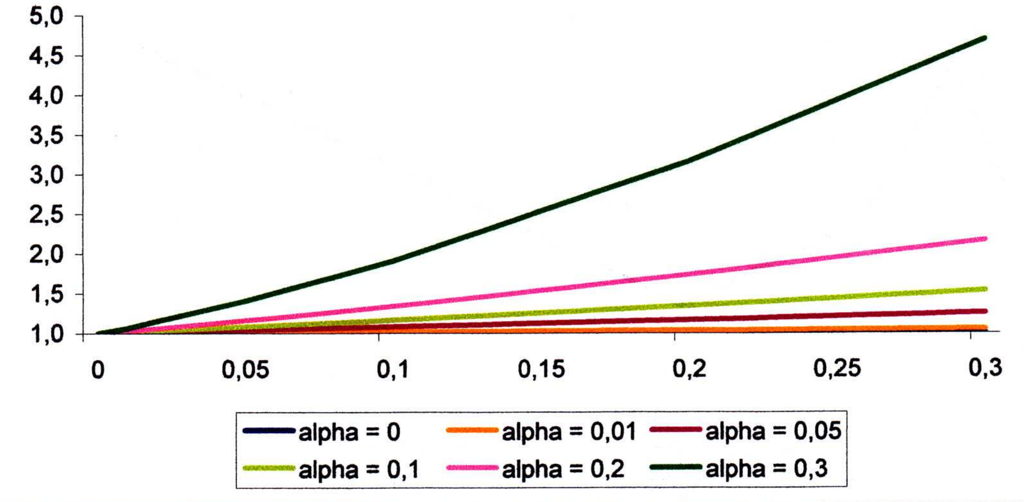


Figure 4: Dependency of empirical relative efficiency of the max$\underline{\mathrm{D}}_\alpha$-estimators with escort parameter $\mu = 0$ on contamination parameter $\varepsilon$ for data distributed by $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$

All that was stated above holds for $\mu = 0$. However, the situation changes to the worse for the parameter $\mu$ moving to 1. The consistency (cf. Figure 5), efficiency (cf. Figure 7), even the robustness tendencies for $\varepsilon$ increasing (cf. Figure 6) slowly vanish, and we see that apart from the case of $\mu = 0$ the max$\underline{\mathrm{D}}_{\alpha}$-estimators do not possess the useful properties we would desire.

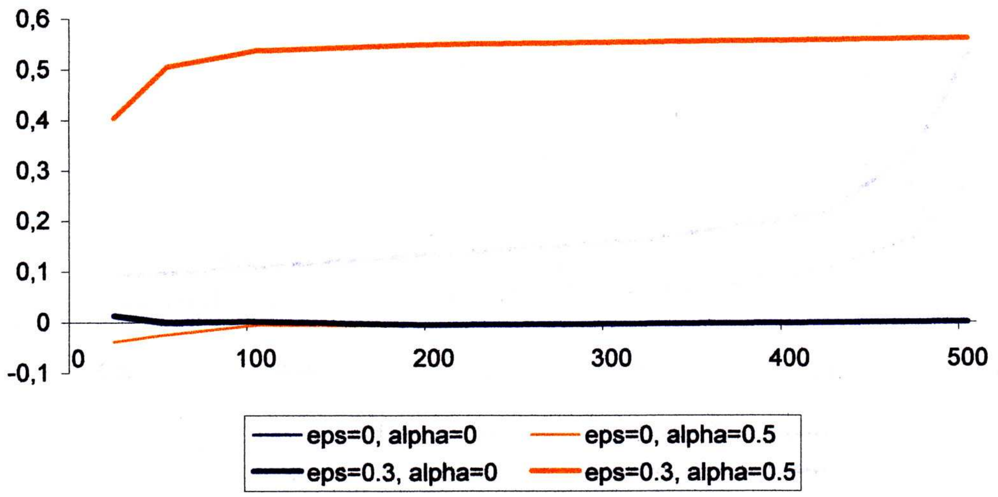


Figure 5: Dependency of expected value of the max$\underline{\mathrm{D}}_{\alpha}$-estimators with escort parameter $\mu = 1$ on sample size $n$ for data distributed by $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$

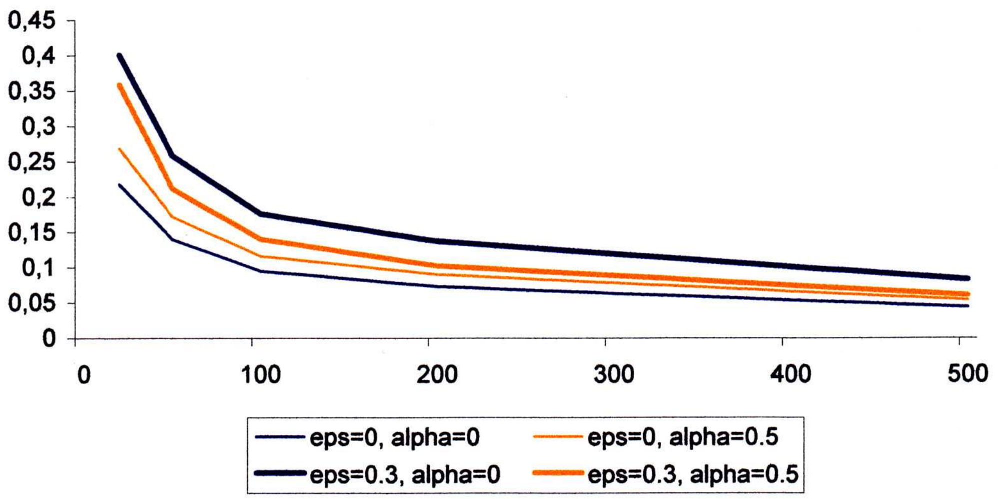


Figure 6: Dependency of standard deviation of the max$\underline{\mathrm{D}}_{\alpha}$-estimators with escort parameter $\mu = 1$ on sample size $n$ for data distributed by $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$

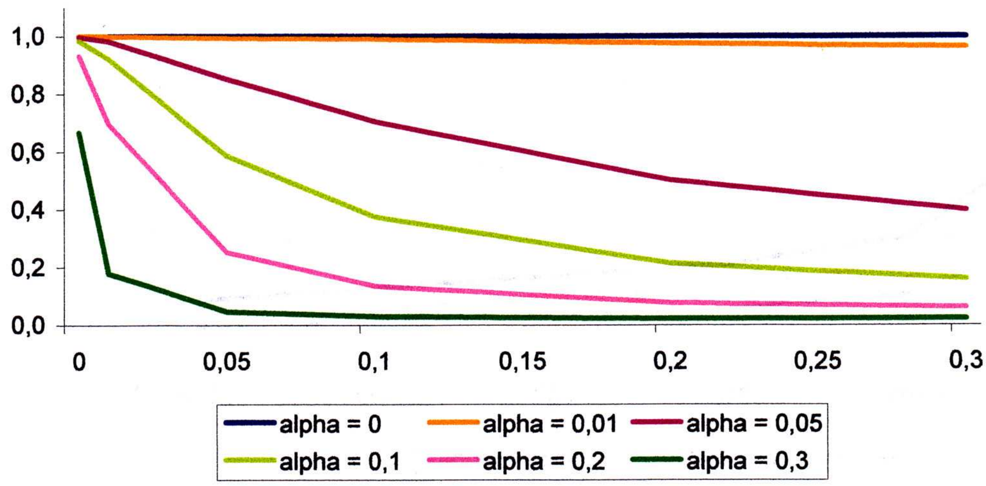


Figure 7: Dependency of empirical relative efficiency of the max$\underline{\mathrm{D}}_{\alpha}$-estimators with escort parameter $\mu = 1$ on contamination parameter $\varepsilon$ for data distributed by $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$

The previously described behavior can be seen also for the other mixtures, i.e. contamination by $N(0,100)$ in the tables $31-60$, $Lo(0,1)$ in the tables 61 - 90 and $C(0,1)$ in the tables $91-120$. It was only observed to grow stronger as the outliers get farther away, as is the case of contamination by Cauchy distribution. Especially the robustness of the estimator escorted by $\mu = 0$ is rather stunning (cf. Table 116) compared to maximum likelihood estimator. Unfortunately also the loss of consistency for $\mu$ moving to 1 is faster.

Finally, we shall examine to what extent the standard deviations obtained for uncontaminated normal model correspond with the asymptotic variance derived in section 3.1, and we see that the standard deviations in tables 1-60 for $\varepsilon = 0$ or $\alpha = 0$ or $\mu = 0$ match with $\sqrt{\mathrm{V}(\tilde{T}_{\alpha,\mu}, P_0)/n}$ almost perfectly. Figure 8 shows the standard deviation and asymptotic variance for $\varepsilon = 0$, escort parameter $\mu = 1$, and power parameter $\alpha = 0.5$ where one of the the highest differences occured.

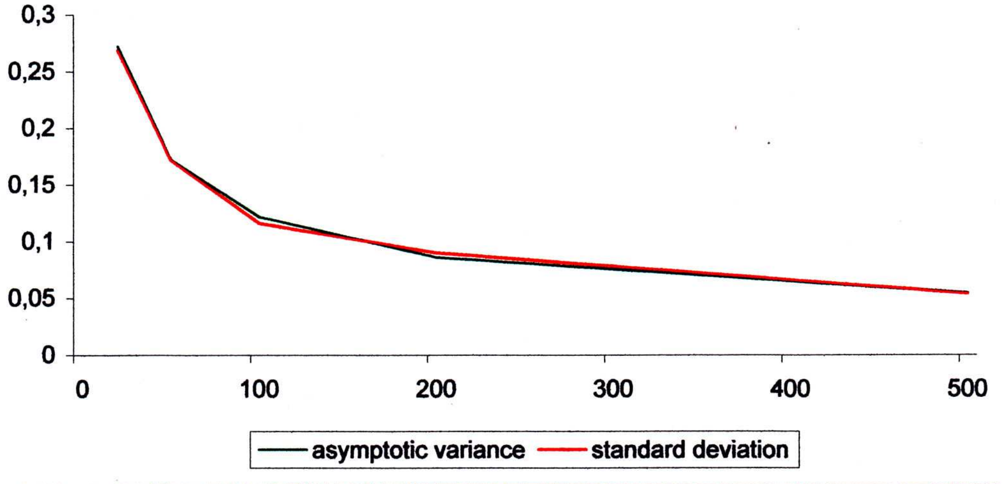


Figure 8: Comparison of standard deviation and asymptotic variance in the normal model N(0,1) for $\mu = 1$ and $\alpha = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.011 | 0.221 | 0.979 | −0.004 | 0.140 | 0.997 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 0.998 |
| 0.20 | 0.011 | 0.224 | 0.953 | −0.004 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.073 | 0.994 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.012 | 0.236 | 0.857 | −0.004 | 0.143 | 0.962 | 0.002 | 0.096 | 0.976 | −0.003 | 0.074 | 0.981 | 0.001 | 0.044 | 0.988 |

Table 1: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.995 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.010 | 0.224 | 0.955 | −0.005 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.990 | 0.001 | 0.044 | 0.997 |
| 0.50 | 0.007 | 0.236 | 0.859 | −0.006 | 0.143 | 0.958 | 0.001 | 0.096 | 0.974 | −0.004 | 0.074 | 0.968 | 0.001 | 0.044 | 0.987 |

Table 2: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.010 | 0.219 | 0.991 | −0.004 | 0.140 | 0.998 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.008 | 0.224 | 0.955 | −0.005 | 0.141 | 0.988 | 0.001 | 0.096 | 0.992 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.002 | 0.237 | 0.855 | −0.008 | 0.144 | 0.943 | 0.000 | 0.097 | 0.962 | −0.004 | 0.075 | 0.948 | 0.001 | 0.044 | 0.976 |

Table 3: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.009 | 0.219 | 0.992 | −0.005 | 0.140 | 0.997 | 0.001 | 0.095 | 0.998 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.008 | 0.221 | 0.980 | −0.005 | 0.141 | 0.992 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.997 |
| 0.20 | 0.004 | 0.225 | 0.947 | −0.007 | 0.142 | 0.972 | 0.001 | 0.096 | 0.978 | −0.004 | 0.075 | 0.960 | 0.001 | 0.044 | 0.983 |
| 0.50 | −0.014 | 0.244 | 0.802 | −0.014 | 0.151 | 0.855 | −0.001 | 0.102 | 0.879 | −0.005 | 0.079 | 0.852 | 0.000 | 0.047 | 0.889 |

Table 4: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.008 | 0.220 | 0.991 | −0.005 | 0.141 | 0.993 | 0.001 | 0.095 | 0.995 | −0.003 | 0.074 | 0.985 | 0.001 | 0.044 | 0.998 |
| 0.10 | 0.004 | 0.222 | 0.972 | −0.007 | 0.142 | 0.977 | 0.001 | 0.096 | 0.980 | −0.003 | 0.075 | 0.963 | 0.001 | 0.044 | 0.985 |
| 0.20 | −0.004 | 0.229 | 0.909 | −0.010 | 0.146 | 0.917 | 0.000 | 0.099 | 0.926 | −0.004 | 0.077 | 0.899 | 0.001 | 0.046 | 0.932 |
| 0.50 | −0.038 | 0.269 | 0.650 | −0.024 | 0.172 | 0.649 | −0.005 | 0.116 | 0.671 | −0.007 | 0.090 | 0.653 | −0.001 | 0.054 | 0.668 |

Table 5: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.01 | 0.011 | 0.226 | 1.000 | −0.004 | 0.144 | 1.002 | 0.002 | 0.099 | 1.002 | −0.002 | 0.077 | 1.002 | 0.002 | 0.045 | 1.001 |
| 0.05 | 0.011 | 0.226 | 1.001 | −0.004 | 0.144 | 1.008 | 0.002 | 0.098 | 1.008 | −0.002 | 0.076 | 1.008 | 0.002 | 0.045 | 1.007 |
| 0.10 | 0.011 | 0.226 | 0.998 | −0.004 | 0.144 | 1.014 | 0.002 | 0.098 | 1.015 | −0.002 | 0.076 | 1.015 | 0.002 | 0.045 | 1.014 |
| 0.20 | 0.011 | 0.227 | 0.986 | −0.004 | 0.143 | 1.023 | 0.002 | 0.097 | 1.027 | −0.003 | 0.076 | 1.029 | 0.002 | 0.045 | 1.028 |
| 0.50 | 0.012 | 0.237 | 0.910 | −0.004 | 0.143 | 1.024 | 0.002 | 0.096 | 1.048 | −0.003 | 0.075 | 1.060 | 0.002 | 0.044 | 1.066 |

Table 6: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.01 | 0.011 | 0.226 | 1.001 | −0.004 | 0.144 | 1.002 | 0.002 | 0.099 | 1.002 | −0.002 | 0.077 | 1.001 | 0.002 | 0.045 | 1.001 |
| 0.05 | 0.011 | 0.226 | 1.001 | −0.003 | 0.144 | 1.008 | 0.002 | 0.098 | 1.008 | −0.002 | 0.076 | 1.007 | 0.002 | 0.045 | 1.007 |
| 0.10 | 0.011 | 0.226 | 0.999 | −0.003 | 0.144 | 1.014 | 0.003 | 0.098 | 1.015 | −0.002 | 0.076 | 1.013 | 0.002 | 0.045 | 1.013 |
| 0.20 | 0.010 | 0.227 | 0.987 | −0.003 | 0.143 | 1.021 | 0.003 | 0.097 | 1.025 | −0.001 | 0.076 | 1.023 | 0.003 | 0.045 | 1.024 |
| 0.50 | 0.008 | 0.237 | 0.910 | −0.003 | 0.144 | 1.015 | 0.005 | 0.097 | 1.040 | 0.000 | 0.075 | 1.041 | 0.005 | 0.044 | 1.045 |

Table 7: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.01 | 0.011 | 0.226 | 1.001 | −0.004 | 0.144 | 1.002 | 0.002 | 0.099 | 1.002 | −0.002 | 0.077 | 1.001 | 0.002 | 0.045 | 1.001 |
| 0.05 | 0.011 | 0.226 | 1.002 | −0.003 | 0.144 | 1.007 | 0.003 | 0.098 | 1.007 | −0.002 | 0.077 | 1.005 | 0.002 | 0.045 | 1.006 |
| 0.10 | 0.010 | 0.226 | 0.999 | −0.003 | 0.144 | 1.012 | 0.003 | 0.098 | 1.013 | −0.001 | 0.076 | 1.010 | 0.003 | 0.045 | 1.010 |
| 0.20 | 0.010 | 0.227 | 0.986 | −0.002 | 0.143 | 1.016 | 0.005 | 0.098 | 1.019 | 0.000 | 0.076 | 1.013 | 0.005 | 0.045 | 1.013 |
| 0.50 | 0.005 | 0.238 | 0.901 | −0.002 | 0.145 | 0.990 | 0.007 | 0.098 | 1.010 | 0.004 | 0.077 | 0.998 | 0.009 | 0.045 | 0.986 |

Table 8: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.01 | 0.011 | 0.226 | 1.001 | −0.003 | 0.145 | 1.002 | 0.002 | 0.099 | 1.001 | −0.002 | 0.077 | 1.001 | 0.002 | 0.045 | 1.001 |
| 0.05 | 0.010 | 0.226 | 1.001 | −0.002 | 0.144 | 1.005 | 0.004 | 0.098 | 1.003 | −0.001 | 0.077 | 1.000 | 0.004 | 0.045 | 1.000 |
| 0.10 | 0.010 | 0.226 | 0.995 | −0.001 | 0.144 | 1.002 | 0.005 | 0.099 | 1.000 | 0.001 | 0.077 | 0.992 | 0.005 | 0.045 | 0.990 |
| 0.20 | 0.008 | 0.229 | 0.969 | 0.000 | 0.146 | 0.981 | 0.009 | 0.100 | 0.974 | 0.005 | 0.078 | 0.957 | 0.009 | 0.046 | 0.939 |
| 0.50 | −0.005 | 0.248 | 0.827 | 0.000 | 0.157 | 0.844 | 0.016 | 0.108 | 0.823 | 0.015 | 0.086 | 0.765 | 0.022 | 0.052 | 0.642 |

Table 9: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.01 | 0.011 | 0.226 | 1.001 | −0.003 | 0.145 | 1.001 | 0.003 | 0.099 | 1.001 | −0.002 | 0.077 | 0.999 | 0.002 | 0.045 | 1.000 |
| 0.05 | 0.010 | 0.226 | 0.997 | −0.001 | 0.145 | 0.995 | 0.006 | 0.099 | 0.991 | 0.001 | 0.077 | 0.984 | 0.005 | 0.046 | 0.983 |
| 0.10 | 0.009 | 0.229 | 0.976 | 0.001 | 0.147 | 0.967 | 0.009 | 0.101 | 0.954 | 0.005 | 0.079 | 0.939 | 0.009 | 0.046 | 0.922 |
| 0.20 | 0.005 | 0.238 | 0.905 | 0.004 | 0.155 | 0.867 | 0.016 | 0.107 | 0.827 | 0.013 | 0.085 | 0.789 | 0.019 | 0.051 | 0.698 |
| 0.50 | −0.021 | 0.282 | 0.638 | 0.006 | 0.197 | 0.541 | 0.033 | 0.143 | 0.450 | 0.040 | 0.124 | 0.345 | 0.056 | 0.092 | 0.179 |

Table 10: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.01 | 0.014 | 0.256 | 1.010 | −0.001 | 0.167 | 1.010 | 0.003 | 0.112 | 1.009 | −0.004 | 0.088 | 1.009 | 0.002 | 0.051 | 1.008 |
| 0.05 | 0.014 | 0.251 | 1.047 | −0.001 | 0.164 | 1.050 | 0.003 | 0.110 | 1.044 | −0.004 | 0.087 | 1.042 | 0.002 | 0.050 | 1.039 |
| 0.10 | 0.014 | 0.247 | 1.082 | −0.001 | 0.160 | 1.095 | 0.003 | 0.108 | 1.087 | −0.004 | 0.085 | 1.083 | 0.002 | 0.049 | 1.079 |
| 0.20 | 0.014 | 0.242 | 1.129 | −0.001 | 0.155 | 1.172 | 0.002 | 0.104 | 1.166 | −0.004 | 0.082 | 1.163 | 0.002 | 0.048 | 1.160 |
| 0.50 | 0.015 | 0.239 | 1.156 | −0.001 | 0.145 | 1.335 | 0.002 | 0.096 | 1.375 | −0.004 | 0.075 | 1.390 | 0.002 | 0.043 | 1.407 |

Table 11: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.01 | 0.014 | 0.256 | 1.011 | −0.000 | 0.167 | 1.010 | 0.003 | 0.112 | 1.009 | −0.004 | 0.088 | 1.009 | 0.002 | 0.051 | 1.007 |
| 0.05 | 0.015 | 0.251 | 1.047 | 0.001 | 0.164 | 1.050 | 0.005 | 0.110 | 1.042 | −0.002 | 0.087 | 1.042 | 0.004 | 0.050 | 1.035 |
| 0.10 | 0.016 | 0.247 | 1.081 | 0.003 | 0.160 | 1.094 | 0.006 | 0.108 | 1.081 | −0.000 | 0.085 | 1.082 | 0.006 | 0.049 | 1.065 |
| 0.20 | 0.018 | 0.242 | 1.127 | 0.005 | 0.155 | 1.167 | 0.009 | 0.105 | 1.150 | 0.003 | 0.082 | 1.155 | 0.009 | 0.048 | 1.116 |
| 0.50 | 0.019 | 0.239 | 1.151 | 0.011 | 0.146 | 1.307 | 0.016 | 0.097 | 1.308 | 0.011 | 0.076 | 1.325 | 0.018 | 0.044 | 1.185 |

Table 12: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.01 | 0.014 | 0.256 | 1.011 | −0.000 | 0.167 | 1.011 | 0.004 | 0.112 | 1.008 | −0.003 | 0.088 | 1.009 | 0.003 | 0.051 | 1.006 |
| 0.05 | 0.016 | 0.251 | 1.046 | 0.003 | 0.164 | 1.049 | 0.006 | 0.111 | 1.039 | −0.000 | 0.087 | 1.041 | 0.006 | 0.050 | 1.026 |
| 0.10 | 0.018 | 0.247 | 1.079 | 0.006 | 0.161 | 1.089 | 0.010 | 0.109 | 1.070 | 0.003 | 0.085 | 1.074 | 0.009 | 0.049 | 1.037 |
| 0.20 | 0.021 | 0.242 | 1.119 | 0.011 | 0.156 | 1.150 | 0.016 | 0.106 | 1.116 | 0.010 | 0.083 | 1.121 | 0.016 | 0.048 | 1.021 |
| 0.50 | 0.024 | 0.242 | 1.123 | 0.022 | 0.150 | 1.226 | 0.030 | 0.100 | 1.159 | 0.026 | 0.079 | 1.137 | 0.034 | 0.045 | 0.821 |

Table 13: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.01 | 0.015 | 0.255 | 1.011 | 0.001 | 0.167 | 1.010 | 0.005 | 0.112 | 1.007 | −0.002 | 0.088 | 1.009 | 0.004 | 0.051 | 1.003 |
| 0.05 | 0.020 | 0.251 | 1.040 | 0.008 | 0.164 | 1.039 | 0.012 | 0.111 | 1.020 | 0.005 | 0.087 | 1.026 | 0.012 | 0.050 | 0.979 |
| 0.10 | 0.025 | 0.249 | 1.059 | 0.016 | 0.162 | 1.054 | 0.021 | 0.110 | 1.009 | 0.014 | 0.087 | 1.014 | 0.021 | 0.050 | 0.889 |
| 0.20 | 0.032 | 0.248 | 1.061 | 0.029 | 0.162 | 1.033 | 0.036 | 0.111 | 0.933 | 0.031 | 0.087 | 0.909 | 0.039 | 0.051 | 0.645 |
| 0.50 | 0.038 | 0.262 | 0.948 | 0.057 | 0.174 | 0.837 | 0.074 | 0.121 | 0.634 | 0.076 | 0.096 | 0.523 | 0.089 | 0.058 | 0.234 |

Table 14: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.01 | 0.017 | 0.255 | 1.010 | 0.003 | 0.167 | 1.009 | 0.007 | 0.112 | 1.003 | −0.000 | 0.088 | 1.007 | 0.006 | 0.051 | 0.994 |
| 0.05 | 0.027 | 0.253 | 1.019 | 0.017 | 0.166 | 1.006 | 0.022 | 0.113 | 0.965 | 0.015 | 0.088 | 0.972 | 0.021 | 0.051 | 0.853 |
| 0.10 | 0.037 | 0.256 | 0.991 | 0.033 | 0.170 | 0.938 | 0.039 | 0.116 | 0.843 | 0.034 | 0.091 | 0.827 | 0.041 | 0.053 | 0.587 |
| 0.20 | 0.050 | 0.268 | 0.888 | 0.062 | 0.186 | 0.735 | 0.073 | 0.131 | 0.569 | 0.071 | 0.103 | 0.501 | 0.081 | 0.062 | 0.252 |
| 0.50 | 0.061 | 0.325 | 0.604 | 0.120 | 0.250 | 0.365 | 0.156 | 0.189 | 0.212 | 0.177 | 0.157 | 0.139 | 0.210 | 0.109 | 0.047 |

Table 15: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.01 | 0.015 | 0.281 | 1.019 | −0.003 | 0.188 | 1.020 | 0.001 | 0.130 | 1.018 | −0.005 | 0.098 | 1.017 | 0.001 | 0.059 | 1.016 |
| 0.05 | 0.015 | 0.272 | 1.087 | −0.002 | 0.181 | 1.097 | 0.001 | 0.125 | 1.090 | −0.005 | 0.095 | 1.084 | 0.001 | 0.057 | 1.081 |
| 0.10 | 0.015 | 0.263 | 1.158 | −0.002 | 0.174 | 1.187 | 0.001 | 0.120 | 1.178 | −0.005 | 0.092 | 1.168 | 0.001 | 0.055 | 1.164 |
| 0.20 | 0.014 | 0.252 | 1.267 | −0.002 | 0.163 | 1.355 | 0.001 | 0.112 | 1.350 | −0.005 | 0.086 | 1.337 | 0.001 | 0.051 | 1.337 |
| 0.50 | 0.013 | 0.238 | 1.422 | −0.001 | 0.142 | 1.779 | 0.001 | 0.096 | 1.850 | −0.004 | 0.073 | 1.863 | 0.001 | 0.043 | 1.908 |

Table 16: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.01 | 0.016 | 0.281 | 1.019 | −0.002 | 0.188 | 1.020 | 0.002 | 0.130 | 1.018 | −0.004 | 0.098 | 1.017 | 0.002 | 0.059 | 1.015 |
| 0.05 | 0.018 | 0.272 | 1.086 | 0.001 | 0.181 | 1.097 | 0.005 | 0.125 | 1.088 | −0.001 | 0.095 | 1.084 | 0.005 | 0.057 | 1.073 |
| 0.10 | 0.019 | 0.264 | 1.154 | 0.005 | 0.174 | 1.187 | 0.008 | 0.120 | 1.172 | 0.002 | 0.092 | 1.164 | 0.009 | 0.055 | 1.136 |
| 0.20 | 0.022 | 0.252 | 1.258 | 0.010 | 0.163 | 1.348 | 0.014 | 0.113 | 1.325 | 0.009 | 0.086 | 1.312 | 0.015 | 0.051 | 1.231 |
| 0.50 | 0.026 | 0.239 | 1.398 | 0.021 | 0.144 | 1.715 | 0.027 | 0.097 | 1.690 | 0.023 | 0.074 | 1.646 | 0.029 | 0.043 | 1.295 |

Table 17: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.01 | 0.016 | 0.281 | 1.019 | −0.001 | 0.188 | 1.020 | 0.003 | 0.129 | 1.018 | −0.004 | 0.098 | 1.017 | 0.003 | 0.058 | 1.014 |
| 0.05 | 0.020 | 0.272 | 1.083 | 0.005 | 0.181 | 1.096 | 0.009 | 0.125 | 1.084 | 0.003 | 0.095 | 1.080 | 0.009 | 0.057 | 1.055 |
| 0.10 | 0.024 | 0.264 | 1.147 | 0.011 | 0.175 | 1.179 | 0.015 | 0.121 | 1.154 | 0.010 | 0.092 | 1.143 | 0.016 | 0.055 | 1.069 |
| 0.20 | 0.030 | 0.253 | 1.239 | 0.022 | 0.164 | 1.314 | 0.027 | 0.114 | 1.255 | 0.022 | 0.087 | 1.218 | 0.028 | 0.051 | 1.008 |
| 0.50 | 0.038 | 0.243 | 1.336 | 0.043 | 0.148 | 1.527 | 0.052 | 0.100 | 1.343 | 0.050 | 0.077 | 1.173 | 0.057 | 0.044 | 0.666 |

Table 18: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.01 | 0.018 | 0.281 | 1.018 | 0.001 | 0.188 | 1.021 | 0.005 | 0.129 | 1.017 | −0.001 | 0.098 | 1.017 | 0.005 | 0.059 | 1.008 |
| 0.05 | 0.028 | 0.273 | 1.070 | 0.016 | 0.182 | 1.082 | 0.020 | 0.126 | 1.053 | 0.014 | 0.096 | 1.042 | 0.020 | 0.057 | 0.950 |
| 0.10 | 0.038 | 0.267 | 1.107 | 0.031 | 0.177 | 1.117 | 0.037 | 0.123 | 1.041 | 0.031 | 0.094 | 0.993 | 0.038 | 0.056 | 0.766 |
| 0.20 | 0.053 | 0.262 | 1.128 | 0.058 | 0.172 | 1.095 | 0.066 | 0.120 | 0.912 | 0.062 | 0.093 | 0.781 | 0.070 | 0.055 | 0.442 |
| 0.50 | 0.075 | 0.270 | 1.025 | 0.111 | 0.175 | 0.841 | 0.131 | 0.122 | 0.535 | 0.135 | 0.096 | 0.360 | 0.147 | 0.055 | 0.140 |

Table 19: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.01 | 0.021 | 0.281 | 1.015 | 0.005 | 0.188 | 1.019 | 0.009 | 0.130 | 1.013 | 0.003 | 0.099 | 1.013 | 0.009 | 0.059 | 0.991 |
| 0.05 | 0.042 | 0.277 | 1.030 | 0.034 | 0.185 | 1.023 | 0.039 | 0.128 | 0.955 | 0.033 | 0.098 | 0.914 | 0.039 | 0.058 | 0.706 |
| 0.10 | 0.062 | 0.278 | 0.992 | 0.066 | 0.187 | 0.920 | 0.073 | 0.130 | 0.763 | 0.069 | 0.102 | 0.654 | 0.076 | 0.060 | 0.375 |
| 0.20 | 0.092 | 0.292 | 0.862 | 0.119 | 0.202 | 0.658 | 0.134 | 0.144 | 0.441 | 0.135 | 0.114 | 0.314 | 0.146 | 0.068 | 0.135 |
| 0.50 | 0.134 | 0.353 | 0.566 | 0.228 | 0.261 | 0.301 | 0.274 | 0.193 | 0.152 | 0.299 | 0.155 | 0.087 | 0.329 | 0.096 | 0.030 |

Table 20: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.01 | 0.017 | 0.344 | 1.037 | −0.001 | 0.224 | 1.037 | 0.002 | 0.154 | 1.035 | −0.005 | 0.118 | 1.033 | 0.001 | 0.071 | 1.032 |
| 0.05 | 0.016 | 0.323 | 1.176 | −0.001 | 0.210 | 1.185 | 0.002 | 0.145 | 1.176 | −0.004 | 0.111 | 1.168 | 0.001 | 0.066 | 1.166 |
| 0.10 | 0.016 | 0.303 | 1.333 | −0.001 | 0.195 | 1.368 | 0.002 | 0.135 | 1.356 | −0.004 | 0.104 | 1.344 | 0.001 | 0.062 | 1.344 |
| 0.20 | 0.015 | 0.276 | 1.606 | −0.002 | 0.173 | 1.732 | 0.002 | 0.119 | 1.731 | −0.004 | 0.092 | 1.721 | 0.001 | 0.054 | 1.732 |
| 0.50 | 0.014 | 0.236 | 2.196 | −0.002 | 0.135 | 2.835 | 0.002 | 0.091 | 2.981 | −0.003 | 0.069 | 3.054 | 0.001 | 0.040 | 3.161 |

Table 21: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,9)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.01 | 0.018 | 0.344 | 1.037 | 0.001 | 0.224 | 1.037 | 0.004 | 0.154 | 1.035 | −0.003 | 0.118 | 1.033 | 0.003 | 0.071 | 1.031 |
| 0.05 | 0.022 | 0.323 | 1.174 | 0.006 | 0.210 | 1.182 | 0.009 | 0.145 | 1.171 | 0.003 | 0.111 | 1.166 | 0.009 | 0.066 | 1.148 |
| 0.10 | 0.027 | 0.303 | 1.327 | 0.011 | 0.195 | 1.358 | 0.015 | 0.135 | 1.338 | 0.009 | 0.104 | 1.329 | 0.015 | 0.062 | 1.271 |
| 0.20 | 0.033 | 0.276 | 1.588 | 0.020 | 0.174 | 1.694 | 0.025 | 0.120 | 1.654 | 0.020 | 0.092 | 1.630 | 0.025 | 0.055 | 1.429 |
| 0.50 | 0.042 | 0.237 | 2.122 | 0.036 | 0.138 | 2.568 | 0.044 | 0.091 | 2.409 | 0.040 | 0.070 | 2.244 | 0.044 | 0.041 | 1.418 |

Table 22: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,9)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.01 | 0.019 | 0.344 | 1.037 | 0.002 | 0.224 | 1.037 | 0.005 | 0.154 | 1.034 | −0.001 | 0.118 | 1.033 | 0.004 | 0.071 | 1.029 |
| 0.05 | 0.028 | 0.323 | 1.171 | 0.013 | 0.210 | 1.175 | 0.017 | 0.145 | 1.159 | 0.010 | 0.112 | 1.153 | 0.016 | 0.066 | 1.103 |
| 0.10 | 0.037 | 0.304 | 1.315 | 0.024 | 0.196 | 1.333 | 0.029 | 0.135 | 1.291 | 0.023 | 0.104 | 1.267 | 0.028 | 0.062 | 1.106 |
| 0.20 | 0.050 | 0.278 | 1.546 | 0.042 | 0.176 | 1.593 | 0.049 | 0.120 | 1.466 | 0.043 | 0.093 | 1.371 | 0.049 | 0.055 | 0.952 |
| 0.50 | 0.070 | 0.241 | 1.946 | 0.075 | 0.142 | 2.015 | 0.086 | 0.094 | 1.536 | 0.083 | 0.072 | 1.202 | 0.088 | 0.042 | 0.538 |

Table 23: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,9)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.01 | 0.023 | 0.344 | 1.036 | 0.007 | 0.224 | 1.034 | 0.010 | 0.154 | 1.031 | 0.003 | 0.118 | 1.031 | 0.009 | 0.071 | 1.016 |
| 0.05 | 0.046 | 0.324 | 1.150 | 0.034 | 0.212 | 1.134 | 0.039 | 0.146 | 1.085 | 0.032 | 0.112 | 1.057 | 0.038 | 0.067 | 0.873 |
| 0.10 | 0.069 | 0.307 | 1.242 | 0.062 | 0.200 | 1.183 | 0.070 | 0.138 | 1.036 | 0.064 | 0.107 | 0.935 | 0.069 | 0.063 | 0.584 |
| 0.20 | 0.101 | 0.288 | 1.321 | 0.107 | 0.186 | 1.128 | 0.119 | 0.127 | 0.814 | 0.115 | 0.099 | 0.628 | 0.121 | 0.058 | 0.284 |
| 0.50 | 0.155 | 0.273 | 1.249 | 0.191 | 0.170 | 0.799 | 0.214 | 0.110 | 0.427 | 0.215 | 0.085 | 0.270 | 0.224 | 0.049 | 0.098 |

Table 24: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,9)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.01 | 0.030 | 0.344 | 1.032 | 0.014 | 0.225 | 1.028 | 0.018 | 0.155 | 1.019 | 0.011 | 0.119 | 1.019 | 0.017 | 0.071 | 0.976 |
| 0.05 | 0.076 | 0.328 | 1.084 | 0.069 | 0.216 | 1.009 | 0.076 | 0.149 | 0.885 | 0.069 | 0.115 | 0.801 | 0.075 | 0.068 | 0.503 |
| 0.10 | 0.121 | 0.321 | 1.043 | 0.127 | 0.214 | 0.841 | 0.138 | 0.147 | 0.604 | 0.133 | 0.115 | 0.468 | 0.139 | 0.068 | 0.214 |
| 0.20 | 0.187 | 0.325 | 0.875 | 0.219 | 0.220 | 0.540 | 0.241 | 0.152 | 0.304 | 0.240 | 0.119 | 0.202 | 0.249 | 0.070 | 0.077 |
| 0.50 | 0.291 | 0.369 | 0.557 | 0.388 | 0.250 | 0.244 | 0.439 | 0.166 | 0.112 | 0.454 | 0.125 | 0.065 | 0.473 | 0.072 | 0.022 |

Table 25: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,9)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.01 | 0.014 | 0.391 | 1.052 | 0.001 | 0.252 | 1.053 | 0.001 | 0.171 | 1.051 | −0.005 | 0.134 | 1.049 | 0.001 | 0.081 | 1.048 |
| 0.05 | 0.014 | 0.359 | 1.250 | 0.001 | 0.230 | 1.265 | 0.001 | 0.157 | 1.261 | −0.005 | 0.122 | 1.252 | 0.001 | 0.074 | 1.253 |
| 0.10 | 0.013 | 0.329 | 1.484 | 0.000 | 0.209 | 1.539 | 0.001 | 0.142 | 1.538 | −0.004 | 0.111 | 1.525 | 0.001 | 0.067 | 1.531 |
| 0.20 | 0.012 | 0.289 | 1.923 | 0.000 | 0.178 | 2.117 | 0.001 | 0.120 | 2.147 | −0.004 | 0.094 | 2.137 | 0.000 | 0.056 | 2.165 |
| 0.50 | 0.010 | 0.229 | 3.060 | 0.000 | 0.128 | 4.089 | 0.001 | 0.084 | 4.402 | −0.003 | 0.064 | 4.502 | 0.000 | 0.038 | 4.692 |

Table 26: $\mathrm{Max}\underset{\sim}{\mathrm{D}}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.01 | 0.016 | 0.391 | 1.052 | 0.003 | 0.252 | 1.052 | 0.004 | 0.171 | 1.050 | −0.003 | 0.134 | 1.049 | 0.003 | 0.081 | 1.047 |
| 0.05 | 0.023 | 0.359 | 1.248 | 0.011 | 0.230 | 1.262 | 0.012 | 0.157 | 1.252 | 0.006 | 0.122 | 1.248 | 0.011 | 0.074 | 1.225 |
| 0.10 | 0.029 | 0.329 | 1.476 | 0.019 | 0.209 | 1.524 | 0.020 | 0.142 | 1.505 | 0.014 | 0.111 | 1.495 | 0.020 | 0.067 | 1.408 |
| 0.20 | 0.037 | 0.289 | 1.893 | 0.030 | 0.178 | 2.047 | 0.032 | 0.120 | 1.992 | 0.028 | 0.094 | 1.951 | 0.032 | 0.056 | 1.621 |
| 0.50 | 0.051 | 0.230 | 2.901 | 0.050 | 0.129 | 3.493 | 0.053 | 0.084 | 3.108 | 0.050 | 0.065 | 2.764 | 0.054 | 0.038 | 1.555 |

Table 27: $\mathrm{Max}\underset{\sim}{\mathrm{D}}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.01 | 0.018 | 0.391 | 1.051 | 0.005 | 0.252 | 1.052 | 0.006 | 0.171 | 1.049 | −0.001 | 0.134 | 1.049 | 0.005 | 0.081 | 1.044 |
| 0.05 | 0.032 | 0.359 | 1.242 | 0.021 | 0.230 | 1.252 | 0.023 | 0.157 | 1.232 | 0.016 | 0.123 | 1.226 | 0.022 | 0.074 | 1.152 |
| 0.10 | 0.045 | 0.330 | 1.457 | 0.037 | 0.209 | 1.483 | 0.039 | 0.142 | 1.419 | 0.033 | 0.112 | 1.384 | 0.039 | 0.067 | 1.142 |
| 0.20 | 0.063 | 0.291 | 1.824 | 0.060 | 0.180 | 1.866 | 0.064 | 0.121 | 1.647 | 0.059 | 0.095 | 1.496 | 0.065 | 0.057 | 0.930 |
| 0.50 | 0.091 | 0.235 | 2.551 | 0.100 | 0.132 | 2.440 | 0.106 | 0.086 | 1.660 | 0.104 | 0.067 | 1.233 | 0.108 | 0.039 | 0.518 |

Table 28: $\mathrm{Max}\underset{\sim}{\mathrm{D}}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.01 | 0.025 | 0.391 | 1.050 | 0.012 | 0.252 | 1.049 | 0.013 | 0.172 | 1.044 | 0.006 | 0.134 | 1.045 | 0.012 | 0.081 | 1.025 |
| 0.05 | 0.059 | 0.360 | 1.212 | 0.052 | 0.232 | 1.189 | 0.054 | 0.158 | 1.109 | 0.048 | 0.124 | 1.067 | 0.054 | 0.075 | 0.816 |
| 0.10 | 0.092 | 0.334 | 1.346 | 0.092 | 0.213 | 1.246 | 0.096 | 0.145 | 1.019 | 0.091 | 0.114 | 0.885 | 0.096 | 0.068 | 0.492 |
| 0.20 | 0.138 | 0.302 | 1.468 | 0.151 | 0.188 | 1.152 | 0.159 | 0.128 | 0.745 | 0.155 | 0.100 | 0.550 | 0.161 | 0.060 | 0.232 |
| 0.50 | 0.210 | 0.265 | 1.410 | 0.250 | 0.152 | 0.782 | 0.265 | 0.099 | 0.387 | 0.266 | 0.075 | 0.245 | 0.273 | 0.044 | 0.090 |

Table 29: $\mathrm{Max}\underset{\sim}{\mathrm{D}}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.01 | 0.035 | 0.391 | 1.045 | 0.024 | 0.253 | 1.040 | 0.025 | 0.172 | 1.025 | 0.018 | 0.134 | 1.025 | 0.024 | 0.081 | 0.963 |
| 0.05 | 0.104 | 0.365 | 1.117 | 0.104 | 0.236 | 1.008 | 0.108 | 0.161 | 0.820 | 0.101 | 0.127 | 0.715 | 0.107 | 0.076 | 0.399 |
| 0.10 | 0.170 | 0.350 | 1.066 | 0.184 | 0.226 | 0.790 | 0.192 | 0.155 | 0.506 | 0.187 | 0.122 | 0.376 | 0.194 | 0.073 | 0.160 |
| 0.20 | 0.263 | 0.340 | 0.872 | 0.304 | 0.217 | 0.479 | 0.320 | 0.150 | 0.247 | 0.320 | 0.117 | 0.162 | 0.328 | 0.070 | 0.061 |
| 0.50 | 0.404 | 0.359 | 0.553 | 0.505 | 0.212 | 0.223 | 0.537 | 0.140 | 0.100 | 0.549 | 0.102 | 0.060 | 0.561 | 0.061 | 0.022 |

Table 30: $\mathrm{Max}\underset{\sim}{\mathrm{D}}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.011 | 0.221 | 0.979 | −0.004 | 0.140 | 0.997 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 0.998 |
| 0.20 | 0.011 | 0.224 | 0.953 | −0.004 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.073 | 0.994 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.012 | 0.236 | 0.857 | −0.004 | 0.143 | 0.962 | 0.002 | 0.096 | 0.976 | −0.003 | 0.074 | 0.981 | 0.001 | 0.044 | 0.988 |

Table 31: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.995 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.010 | 0.224 | 0.955 | −0.005 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.990 | 0.001 | 0.044 | 0.997 |
| 0.50 | 0.007 | 0.236 | 0.859 | −0.006 | 0.143 | 0.958 | 0.001 | 0.096 | 0.974 | −0.004 | 0.074 | 0.968 | 0.001 | 0.044 | 0.987 |

Table 32: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.010 | 0.219 | 0.991 | −0.004 | 0.140 | 0.998 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.008 | 0.224 | 0.955 | −0.005 | 0.141 | 0.988 | 0.001 | 0.096 | 0.992 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.002 | 0.237 | 0.855 | −0.008 | 0.144 | 0.943 | 0.000 | 0.097 | 0.962 | −0.004 | 0.075 | 0.948 | 0.001 | 0.044 | 0.976 |

Table 33: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.009 | 0.219 | 0.992 | −0.005 | 0.140 | 0.997 | 0.001 | 0.095 | 0.998 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.008 | 0.221 | 0.980 | −0.005 | 0.141 | 0.992 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.997 |
| 0.20 | 0.004 | 0.225 | 0.947 | −0.007 | 0.142 | 0.972 | 0.001 | 0.096 | 0.978 | −0.004 | 0.075 | 0.960 | 0.001 | 0.044 | 0.983 |
| 0.50 | −0.014 | 0.244 | 0.802 | −0.014 | 0.151 | 0.855 | −0.001 | 0.102 | 0.879 | −0.005 | 0.079 | 0.852 | 0.000 | 0.047 | 0.889 |

Table 34: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.008 | 0.220 | 0.991 | −0.005 | 0.141 | 0.993 | 0.001 | 0.095 | 0.995 | −0.003 | 0.074 | 0.985 | 0.001 | 0.044 | 0.998 |
| 0.10 | 0.004 | 0.222 | 0.972 | −0.007 | 0.142 | 0.977 | 0.001 | 0.096 | 0.980 | −0.003 | 0.075 | 0.963 | 0.001 | 0.044 | 0.985 |
| 0.20 | −0.004 | 0.229 | 0.909 | −0.010 | 0.146 | 0.917 | 0.000 | 0.099 | 0.926 | −0.004 | 0.077 | 0.899 | 0.001 | 0.046 | 0.932 |
| 0.50 | −0.038 | 0.269 | 0.650 | −0.024 | 0.172 | 0.649 | −0.005 | 0.116 | 0.671 | −0.007 | 0.090 | 0.653 | −0.001 | 0.054 | 0.668 |

Table 35: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.01 | 0.010 | 0.280 | 1.093 | −0.002 | 0.186 | 1.061 | 0.003 | 0.132 | 1.048 | −0.001 | 0.103 | 1.033 | 0.002 | 0.060 | 1.025 |
| 0.05 | 0.010 | 0.257 | 1.294 | −0.002 | 0.173 | 1.233 | 0.003 | 0.123 | 1.197 | −0.001 | 0.097 | 1.149 | 0.002 | 0.058 | 1.119 |
| 0.10 | 0.010 | 0.246 | 1.417 | −0.003 | 0.164 | 1.371 | 0.003 | 0.116 | 1.339 | −0.001 | 0.093 | 1.272 | 0.002 | 0.055 | 1.230 |
| 0.20 | 0.010 | 0.237 | 1.524 | −0.003 | 0.154 | 1.543 | 0.003 | 0.108 | 1.547 | −0.001 | 0.086 | 1.477 | 0.002 | 0.051 | 1.436 |
| 0.50 | 0.010 | 0.237 | 1.520 | −0.004 | 0.145 | 1.757 | 0.003 | 0.097 | 1.915 | −0.001 | 0.075 | 1.916 | 0.001 | 0.043 | 1.984 |

Table 36: $\mathrm{Max}\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,100)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.01 | 0.011 | 0.280 | 1.093 | −0.001 | 0.186 | 1.060 | 0.004 | 0.132 | 1.047 | 0.000 | 0.103 | 1.032 | 0.003 | 0.060 | 1.022 |
| 0.05 | 0.012 | 0.257 | 1.293 | 0.001 | 0.173 | 1.225 | 0.007 | 0.123 | 1.190 | 0.003 | 0.098 | 1.139 | 0.007 | 0.058 | 1.095 |
| 0.10 | 0.012 | 0.246 | 1.413 | 0.002 | 0.165 | 1.355 | 0.009 | 0.117 | 1.318 | 0.007 | 0.093 | 1.242 | 0.010 | 0.056 | 1.159 |
| 0.20 | 0.013 | 0.238 | 1.517 | 0.003 | 0.156 | 1.511 | 0.013 | 0.110 | 1.490 | 0.011 | 0.088 | 1.389 | 0.016 | 0.053 | 1.225 |
| 0.50 | 0.011 | 0.238 | 1.508 | 0.005 | 0.148 | 1.683 | 0.019 | 0.100 | 1.736 | 0.020 | 0.080 | 1.599 | 0.029 | 0.048 | 1.197 |

Table 37: $\mathrm{Max}\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,100)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.01 | 0.011 | 0.280 | 1.093 | −0.000 | 0.187 | 1.058 | 0.005 | 0.132 | 1.045 | 0.001 | 0.103 | 1.031 | 0.004 | 0.060 | 1.018 |
| 0.05 | 0.013 | 0.258 | 1.288 | 0.004 | 0.174 | 1.210 | 0.011 | 0.124 | 1.171 | 0.008 | 0.099 | 1.115 | 0.011 | 0.059 | 1.046 |
| 0.10 | 0.015 | 0.247 | 1.402 | 0.007 | 0.167 | 1.320 | 0.016 | 0.119 | 1.261 | 0.014 | 0.096 | 1.168 | 0.019 | 0.058 | 1.015 |
| 0.20 | 0.015 | 0.239 | 1.495 | 0.010 | 0.160 | 1.436 | 0.023 | 0.114 | 1.345 | 0.024 | 0.093 | 1.185 | 0.031 | 0.057 | 0.871 |
| 0.50 | 0.011 | 0.242 | 1.464 | 0.014 | 0.155 | 1.511 | 0.035 | 0.110 | 1.353 | 0.042 | 0.092 | 1.059 | 0.058 | 0.060 | 0.538 |

Table 38: $\mathrm{Max}\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,100)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.01 | 0.013 | 0.280 | 1.090 | 0.002 | 0.187 | 1.050 | 0.008 | 0.132 | 1.037 | 0.004 | 0.103 | 1.022 | 0.007 | 0.061 | 0.999 |
| 0.05 | 0.019 | 0.261 | 1.256 | 0.013 | 0.180 | 1.130 | 0.022 | 0.129 | 1.053 | 0.021 | 0.104 | 0.976 | 0.025 | 0.063 | 0.812 |
| 0.10 | 0.022 | 0.253 | 1.330 | 0.021 | 0.179 | 1.136 | 0.036 | 0.132 | 0.968 | 0.038 | 0.109 | 0.811 | 0.046 | 0.070 | 0.527 |
| 0.20 | 0.023 | 0.250 | 1.357 | 0.031 | 0.182 | 1.078 | 0.055 | 0.142 | 0.783 | 0.066 | 0.124 | 0.548 | 0.083 | 0.089 | 0.251 |
| 0.50 | 0.013 | 0.266 | 1.208 | 0.041 | 0.199 | 0.889 | 0.085 | 0.168 | 0.511 | 0.117 | 0.160 | 0.278 | 0.165 | 0.126 | 0.087 |

Table 39: $\mathrm{Max}\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,100)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.01 | 0.016 | 0.281 | 1.080 | 0.006 | 0.189 | 1.027 | 0.012 | 0.133 | 1.009 | 0.009 | 0.104 | 0.995 | 0.012 | 0.061 | 0.948 |
| 0.05 | 0.027 | 0.271 | 1.153 | 0.028 | 0.198 | 0.919 | 0.043 | 0.148 | 0.763 | 0.045 | 0.121 | 0.652 | 0.051 | 0.079 | 0.423 |
| 0.10 | 0.033 | 0.274 | 1.125 | 0.046 | 0.217 | 0.750 | 0.072 | 0.176 | 0.499 | 0.086 | 0.157 | 0.341 | 0.104 | 0.117 | 0.153 |
| 0.20 | 0.035 | 0.288 | 1.022 | 0.067 | 0.249 | 0.553 | 0.115 | 0.226 | 0.282 | 0.154 | 0.221 | 0.151 | 0.204 | 0.185 | 0.049 |
| 0.50 | 0.014 | 0.337 | 0.754 | 0.088 | 0.313 | 0.348 | 0.179 | 0.312 | 0.140 | 0.273 | 0.322 | 0.061 | 0.408 | 0.273 | 0.015 |

Table 40: $\mathrm{Max}\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon N(0,100)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.01 | 0.024 | 0.468 | 1.248 | 0.011 | 0.319 | 1.181 | 0.007 | 0.221 | 1.138 | −0.008 | 0.171 | 1.119 | 0.004 | 0.098 | 1.106 |
| 0.05 | 0.022 | 0.372 | 1.974 | 0.008 | 0.259 | 1.786 | 0.005 | 0.184 | 1.646 | −0.007 | 0.143 | 1.592 | 0.003 | 0.083 | 1.557 |
| 0.10 | 0.020 | 0.321 | 2.641 | 0.007 | 0.222 | 2.427 | 0.004 | 0.158 | 2.243 | −0.006 | 0.122 | 2.196 | 0.003 | 0.070 | 2.182 |
| 0.20 | 0.018 | 0.276 | 3.582 | 0.005 | 0.184 | 3.525 | 0.003 | 0.128 | 3.393 | −0.005 | 0.097 | 3.476 | 0.002 | 0.054 | 3.647 |
| 0.50 | 0.016 | 0.238 | 4.822 | 0.002 | 0.142 | 5.950 | 0.002 | 0.091 | 6.679 | −0.004 | 0.064 | 7.859 | 0.001 | 0.033 | 9.744 |

Table 41: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.01 | 0.027 | 0.468 | 1.247 | 0.015 | 0.319 | 1.178 | 0.011 | 0.221 | 1.135 | −0.004 | 0.171 | 1.121 | 0.009 | 0.098 | 1.099 |
| 0.05 | 0.033 | 0.372 | 1.961 | 0.023 | 0.260 | 1.761 | 0.022 | 0.185 | 1.611 | 0.011 | 0.144 | 1.576 | 0.022 | 0.083 | 1.443 |
| 0.10 | 0.035 | 0.322 | 2.605 | 0.029 | 0.224 | 2.353 | 0.031 | 0.159 | 2.124 | 0.023 | 0.123 | 2.087 | 0.034 | 0.071 | 1.736 |
| 0.20 | 0.037 | 0.278 | 3.491 | 0.036 | 0.187 | 3.293 | 0.042 | 0.131 | 2.963 | 0.039 | 0.099 | 2.896 | 0.049 | 0.055 | 1.954 |
| 0.50 | 0.038 | 0.241 | 4.606 | 0.044 | 0.148 | 5.038 | 0.056 | 0.096 | 4.490 | 0.059 | 0.068 | 3.996 | 0.069 | 0.035 | 1.782 |

Table 42: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.01 | 0.031 | 0.468 | 1.245 | 0.019 | 0.319 | 1.175 | 0.016 | 0.221 | 1.130 | 0.001 | 0.171 | 1.121 | 0.013 | 0.098 | 1.086 |
| 0.05 | 0.043 | 0.374 | 1.936 | 0.038 | 0.262 | 1.712 | 0.039 | 0.186 | 1.536 | 0.029 | 0.145 | 1.497 | 0.041 | 0.084 | 1.217 |
| 0.10 | 0.050 | 0.325 | 2.530 | 0.052 | 0.228 | 2.194 | 0.058 | 0.163 | 1.870 | 0.053 | 0.126 | 1.754 | 0.066 | 0.073 | 1.112 |
| 0.20 | 0.057 | 0.283 | 3.294 | 0.067 | 0.195 | 2.820 | 0.080 | 0.137 | 2.200 | 0.082 | 0.104 | 1.850 | 0.097 | 0.059 | 0.835 |
| 0.50 | 0.061 | 0.251 | 4.110 | 0.086 | 0.162 | 3.551 | 0.111 | 0.109 | 2.308 | 0.122 | 0.078 | 1.558 | 0.138 | 0.039 | 0.518 |

Table 43: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.01 | 0.041 | 0.469 | 1.236 | 0.032 | 0.320 | 1.157 | 0.029 | 0.223 | 1.104 | 0.015 | 0.172 | 1.104 | 0.027 | 0.099 | 1.017 |
| 0.05 | 0.075 | 0.383 | 1.794 | 0.083 | 0.275 | 1.458 | 0.090 | 0.198 | 1.181 | 0.085 | 0.153 | 1.068 | 0.100 | 0.090 | 0.590 |
| 0.10 | 0.095 | 0.345 | 2.142 | 0.120 | 0.254 | 1.523 | 0.139 | 0.186 | 1.033 | 0.144 | 0.144 | 0.788 | 0.164 | 0.086 | 0.311 |
| 0.20 | 0.114 | 0.317 | 2.413 | 0.162 | 0.240 | 1.432 | 0.198 | 0.178 | 0.784 | 0.219 | 0.136 | 0.494 | 0.247 | 0.079 | 0.159 |
| 0.50 | 0.126 | 0.313 | 2.402 | 0.213 | 0.239 | 1.172 | 0.278 | 0.175 | 0.517 | 0.320 | 0.123 | 0.279 | 0.356 | 0.060 | 0.082 |

Table 44: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.01 | 0.059 | 0.473 | 1.205 | 0.053 | 0.325 | 1.105 | 0.051 | 0.227 | 1.032 | 0.038 | 0.174 | 1.035 | 0.050 | 0.101 | 0.843 |
| 0.05 | 0.128 | 0.417 | 1.438 | 0.160 | 0.315 | 0.958 | 0.178 | 0.234 | 0.644 | 0.181 | 0.182 | 0.496 | 0.204 | 0.112 | 0.198 |
| 0.10 | 0.170 | 0.408 | 1.404 | 0.238 | 0.332 | 0.721 | 0.281 | 0.257 | 0.384 | 0.309 | 0.202 | 0.240 | 0.349 | 0.128 | 0.078 |
| 0.20 | 0.208 | 0.418 | 1.254 | 0.325 | 0.362 | 0.508 | 0.408 | 0.287 | 0.224 | 0.471 | 0.221 | 0.121 | 0.535 | 0.130 | 0.035 |
| 0.50 | 0.232 | 0.471 | 0.994 | 0.430 | 0.415 | 0.336 | 0.575 | 0.321 | 0.128 | 0.681 | 0.216 | 0.064 | 0.759 | 0.096 | 0.018 |

Table 45: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.01 | 0.028 | 0.584 | 1.340 | 0.004 | 0.423 | 1.282 | 0.000 | 0.297 | 1.251 | −0.012 | 0.216 | 1.227 | 0.002 | 0.132 | 1.212 |
| 0.05 | 0.022 | 0.436 | 2.402 | 0.005 | 0.313 | 2.348 | 0.000 | 0.220 | 2.282 | −0.010 | 0.160 | 2.229 | 0.001 | 0.098 | 2.214 |
| 0.10 | 0.018 | 0.361 | 3.504 | 0.005 | 0.250 | 3.678 | 0.000 | 0.173 | 3.690 | −0.008 | 0.124 | 3.724 | 0.001 | 0.075 | 3.816 |
| 0.20 | 0.015 | 0.293 | 5.321 | 0.004 | 0.189 | 6.407 | 0.000 | 0.126 | 6.908 | −0.006 | 0.087 | 7.514 | 0.001 | 0.051 | 8.188 |
| 0.50 | 0.011 | 0.230 | 8.630 | 0.003 | 0.125 | 14.627 | −0.000 | 0.076 | 19.073 | −0.003 | 0.048 | 25.010 | 0.000 | 0.026 | 30.635 |

Table 46: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.01 | 0.035 | 0.585 | 1.337 | 0.012 | 0.423 | 1.282 | 0.009 | 0.297 | 1.250 | −0.003 | 0.216 | 1.229 | 0.010 | 0.132 | 1.206 |
| 0.05 | 0.041 | 0.437 | 2.379 | 0.032 | 0.313 | 2.324 | 0.030 | 0.220 | 2.230 | 0.022 | 0.161 | 2.178 | 0.033 | 0.098 | 1.979 |
| 0.10 | 0.045 | 0.362 | 3.438 | 0.044 | 0.250 | 3.556 | 0.044 | 0.174 | 3.428 | 0.039 | 0.125 | 3.344 | 0.049 | 0.075 | 2.640 |
| 0.20 | 0.049 | 0.295 | 5.125 | 0.056 | 0.191 | 5.821 | 0.059 | 0.128 | 5.562 | 0.057 | 0.089 | 5.156 | 0.066 | 0.051 | 3.059 |
| 0.50 | 0.052 | 0.234 | 7.978 | 0.069 | 0.129 | 10.815 | 0.075 | 0.079 | 9.329 | 0.077 | 0.049 | 6.896 | 0.082 | 0.027 | 2.826 |

Table 47: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.01 | 0.041 | 0.585 | 1.333 | 0.021 | 0.423 | 1.280 | 0.018 | 0.297 | 1.245 | 0.006 | 0.216 | 1.225 | 0.019 | 0.132 | 1.187 |
| 0.05 | 0.060 | 0.439 | 2.335 | 0.059 | 0.314 | 2.252 | 0.060 | 0.222 | 2.090 | 0.053 | 0.162 | 1.968 | 0.066 | 0.098 | 1.511 |
| 0.10 | 0.071 | 0.366 | 3.299 | 0.083 | 0.253 | 3.240 | 0.088 | 0.177 | 2.832 | 0.086 | 0.127 | 2.430 | 0.098 | 0.076 | 1.382 |
| 0.20 | 0.082 | 0.301 | 4.704 | 0.107 | 0.196 | 4.610 | 0.117 | 0.133 | 3.523 | 0.120 | 0.092 | 2.516 | 0.131 | 0.052 | 1.068 |
| 0.50 | 0.093 | 0.246 | 6.639 | 0.134 | 0.139 | 6.169 | 0.150 | 0.086 | 3.695 | 0.157 | 0.053 | 2.099 | 0.165 | 0.027 | 0.760 |

Table 48: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.01 | 0.060 | 0.587 | 1.316 | 0.045 | 0.424 | 1.264 | 0.043 | 0.298 | 1.215 | 0.032 | 0.217 | 1.187 | 0.046 | 0.133 | 1.076 |
| 0.05 | 0.116 | 0.452 | 2.107 | 0.140 | 0.324 | 1.842 | 0.149 | 0.232 | 1.451 | 0.148 | 0.171 | 1.123 | 0.164 | 0.103 | 0.567 |
| 0.10 | 0.150 | 0.390 | 2.631 | 0.200 | 0.274 | 1.996 | 0.220 | 0.195 | 1.273 | 0.228 | 0.142 | 0.797 | 0.246 | 0.083 | 0.315 |
| 0.20 | 0.183 | 0.341 | 3.060 | 0.262 | 0.232 | 1.881 | 0.294 | 0.161 | 0.982 | 0.311 | 0.110 | 0.528 | 0.330 | 0.060 | 0.189 |
| 0.50 | 0.213 | 0.317 | 3.139 | 0.331 | 0.198 | 1.548 | 0.376 | 0.124 | 0.702 | 0.400 | 0.069 | 0.348 | 0.415 | 0.033 | 0.122 |

Table 49: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.01 | 0.092 | 0.594 | 1.271 | 0.087 | 0.428 | 1.204 | 0.087 | 0.302 | 1.117 | 0.077 | 0.221 | 1.048 | 0.091 | 0.134 | 0.807 |
| 0.05 | 0.211 | 0.494 | 1.588 | 0.276 | 0.363 | 1.104 | 0.301 | 0.266 | 0.683 | 0.310 | 0.198 | 0.423 | 0.333 | 0.119 | 0.170 |
| 0.10 | 0.281 | 0.465 | 1.553 | 0.399 | 0.344 | 0.828 | 0.447 | 0.254 | 0.417 | 0.475 | 0.184 | 0.221 | 0.506 | 0.106 | 0.079 |
| 0.20 | 0.348 | 0.458 | 1.384 | 0.524 | 0.336 | 0.593 | 0.599 | 0.238 | 0.265 | 0.645 | 0.155 | 0.130 | 0.681 | 0.082 | 0.045 |
| 0.50 | 0.406 | 0.494 | 1.120 | 0.662 | 0.339 | 0.415 | 0.766 | 0.210 | 0.175 | 0.821 | 0.099 | 0.084 | 0.847 | 0.045 | 0.029 |

Table 50: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.01 | 0.035 | 0.809 | 1.532 | 0.008 | 0.536 | 1.496 | 0.004 | 0.376 | 1.464 | −0.009 | 0.279 | 1.447 | 0.001 | 0.171 | 1.440 |
| 0.05 | 0.027 | 0.534 | 3.510 | 0.004 | 0.339 | 3.747 | 0.003 | 0.234 | 3.768 | −0.006 | 0.172 | 3.828 | 0.001 | 0.104 | 3.912 |
| 0.10 | 0.022 | 0.409 | 5.989 | 0.002 | 0.244 | 7.224 | 0.002 | 0.164 | 7.664 | −0.005 | 0.118 | 8.166 | 0.001 | 0.070 | 8.616 |
| 0.20 | 0.016 | 0.302 | 10.986 | 0.000 | 0.164 | 16.093 | 0.001 | 0.105 | 18.681 | −0.003 | 0.073 | 21.374 | 0.000 | 0.042 | 23.418 |
| 0.50 | 0.011 | 0.203 | 24.349 | −0.001 | 0.089 | 53.934 | 0.001 | 0.052 | 75.904 | −0.001 | 0.034 | 97.153 | 0.000 | 0.019 | 110.977 |

Table 51: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.01 | 0.048 | 0.808 | 1.530 | 0.023 | 0.537 | 1.492 | 0.020 | 0.376 | 1.460 | 0.007 | 0.280 | 1.445 | 0.017 | 0.171 | 1.425 |
| 0.05 | 0.062 | 0.534 | 3.470 | 0.048 | 0.340 | 3.658 | 0.050 | 0.234 | 3.601 | 0.042 | 0.172 | 3.597 | 0.050 | 0.104 | 3.177 |
| 0.10 | 0.069 | 0.409 | 5.828 | 0.061 | 0.245 | 6.738 | 0.065 | 0.165 | 6.608 | 0.060 | 0.118 | 6.434 | 0.066 | 0.070 | 4.533 |
| 0.20 | 0.074 | 0.303 | 10.330 | 0.072 | 0.165 | 13.218 | 0.078 | 0.106 | 12.011 | 0.075 | 0.073 | 10.251 | 0.079 | 0.042 | 5.167 |
| 0.50 | 0.078 | 0.205 | 20.787 | 0.084 | 0.092 | 27.742 | 0.089 | 0.053 | 19.239 | 0.088 | 0.034 | 12.558 | 0.091 | 0.019 | 4.882 |

Table 52: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.01 | 0.061 | 0.808 | 1.527 | 0.038 | 0.537 | 1.485 | 0.036 | 0.376 | 1.449 | 0.023 | 0.280 | 1.434 | 0.034 | 0.171 | 1.386 |
| 0.05 | 0.098 | 0.535 | 3.387 | 0.091 | 0.341 | 3.445 | 0.097 | 0.235 | 3.195 | 0.090 | 0.173 | 2.966 | 0.099 | 0.104 | 2.042 |
| 0.10 | 0.116 | 0.412 | 5.484 | 0.119 | 0.248 | 5.680 | 0.128 | 0.166 | 4.715 | 0.125 | 0.119 | 3.796 | 0.132 | 0.070 | 1.880 |
| 0.20 | 0.132 | 0.307 | 8.982 | 0.145 | 0.169 | 8.711 | 0.155 | 0.107 | 5.854 | 0.154 | 0.074 | 3.886 | 0.159 | 0.043 | 1.552 |
| 0.50 | 0.146 | 0.214 | 14.901 | 0.169 | 0.097 | 11.354 | 0.178 | 0.054 | 5.976 | 0.178 | 0.035 | 3.423 | 0.181 | 0.020 | 1.264 |

Table 53: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.01 | 0.101 | 0.810 | 1.507 | 0.084 | 0.539 | 1.447 | 0.085 | 0.377 | 1.384 | 0.072 | 0.281 | 1.342 | 0.083 | 0.171 | 1.162 |
| 0.05 | 0.204 | 0.546 | 2.950 | 0.223 | 0.351 | 2.481 | 0.239 | 0.241 | 1.795 | 0.235 | 0.179 | 1.296 | 0.246 | 0.106 | 0.582 |
| 0.10 | 0.257 | 0.431 | 3.978 | 0.297 | 0.263 | 2.734 | 0.318 | 0.174 | 1.573 | 0.320 | 0.126 | 0.960 | 0.330 | 0.073 | 0.368 |
| 0.20 | 0.304 | 0.340 | 4.824 | 0.361 | 0.189 | 2.586 | 0.385 | 0.116 | 1.277 | 0.389 | 0.080 | 0.716 | 0.397 | 0.045 | 0.262 |
| 0.50 | 0.348 | 0.273 | 5.129 | 0.423 | 0.124 | 2.215 | 0.445 | 0.062 | 1.026 | 0.449 | 0.038 | 0.558 | 0.453 | 0.021 | 0.204 |

Table 54: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.01 | 0.166 | 0.815 | 1.449 | 0.161 | 0.545 | 1.332 | 0.166 | 0.382 | 1.195 | 0.153 | 0.285 | 1.080 | 0.165 | 0.173 | 0.736 |
| 0.05 | 0.383 | 0.586 | 2.051 | 0.445 | 0.384 | 1.244 | 0.478 | 0.263 | 0.695 | 0.481 | 0.195 | 0.420 | 0.496 | 0.115 | 0.162 |
| 0.10 | 0.493 | 0.499 | 2.040 | 0.594 | 0.311 | 0.956 | 0.639 | 0.202 | 0.461 | 0.649 | 0.144 | 0.256 | 0.665 | 0.082 | 0.093 |
| 0.20 | 0.590 | 0.442 | 1.846 | 0.725 | 0.247 | 0.733 | 0.775 | 0.144 | 0.333 | 0.788 | 0.095 | 0.180 | 0.801 | 0.051 | 0.065 |
| 0.50 | 0.679 | 0.426 | 1.564 | 0.848 | 0.192 | 0.569 | 0.892 | 0.084 | 0.257 | 0.903 | 0.047 | 0.138 | 0.910 | 0.024 | 0.051 |

Table 55: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.01 | 0.023 | 0.935 | 1.699 | 0.015 | 0.596 | 1.710 | 0.001 | 0.410 | 1.692 | −0.011 | 0.313 | 1.677 | −0.001 | 0.195 | 1.680 |
| 0.05 | 0.016 | 0.564 | 4.667 | 0.009 | 0.333 | 5.475 | 0.000 | 0.223 | 5.719 | −0.006 | 0.168 | 5.837 | −0.001 | 0.103 | 6.041 |
| 0.10 | 0.012 | 0.408 | 8.925 | 0.006 | 0.223 | 12.211 | 0.000 | 0.145 | 13.566 | −0.004 | 0.107 | 14.323 | −0.000 | 0.065 | 15.194 |
| 0.20 | 0.008 | 0.281 | 18.817 | 0.003 | 0.138 | 31.894 | 0.000 | 0.086 | 38.492 | −0.002 | 0.062 | 42.234 | −0.000 | 0.037 | 45.849 |
| 0.50 | 0.003 | 0.167 | 53.389 | 0.001 | 0.067 | 135.225 | −0.000 | 0.039 | 186.525 | −0.001 | 0.028 | 213.451 | −0.000 | 0.016 | 236.565 |

Table 56: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.01 | 0.042 | 0.935 | 1.697 | 0.036 | 0.596 | 1.705 | 0.023 | 0.410 | 1.686 | 0.011 | 0.313 | 1.675 | 0.021 | 0.195 | 1.661 |
| 0.05 | 0.063 | 0.564 | 4.610 | 0.064 | 0.333 | 5.277 | 0.058 | 0.223 | 5.345 | 0.052 | 0.168 | 5.312 | 0.059 | 0.103 | 4.558 |
| 0.10 | 0.072 | 0.409 | 8.642 | 0.076 | 0.223 | 10.918 | 0.073 | 0.145 | 10.798 | 0.069 | 0.107 | 10.058 | 0.074 | 0.065 | 6.600 |
| 0.20 | 0.079 | 0.282 | 17.327 | 0.085 | 0.139 | 22.999 | 0.084 | 0.086 | 19.651 | 0.082 | 0.063 | 15.407 | 0.085 | 0.037 | 7.400 |
| 0.50 | 0.085 | 0.170 | 41.238 | 0.093 | 0.068 | 46.124 | 0.093 | 0.039 | 28.051 | 0.092 | 0.028 | 17.770 | 0.093 | 0.016 | 7.089 |

Table 57: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.01 | 0.061 | 0.935 | 1.693 | 0.058 | 0.596 | 1.694 | 0.046 | 0.411 | 1.668 | 0.034 | 0.314 | 1.655 | 0.044 | 0.195 | 1.598 |
| 0.05 | 0.110 | 0.566 | 4.477 | 0.120 | 0.334 | 4.829 | 0.116 | 0.224 | 4.476 | 0.111 | 0.169 | 4.042 | 0.118 | 0.103 | 2.607 |
| 0.10 | 0.132 | 0.411 | 7.991 | 0.146 | 0.224 | 8.467 | 0.146 | 0.146 | 6.703 | 0.143 | 0.108 | 5.121 | 0.148 | 0.065 | 2.435 |
| 0.20 | 0.150 | 0.285 | 14.283 | 0.167 | 0.140 | 12.821 | 0.168 | 0.087 | 7.961 | 0.167 | 0.063 | 5.167 | 0.170 | 0.037 | 2.099 |
| 0.50 | 0.166 | 0.177 | 25.234 | 0.184 | 0.069 | 15.725 | 0.186 | 0.040 | 7.904 | 0.185 | 0.028 | 4.683 | 0.187 | 0.017 | 1.811 |

Table 58: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.01 | 0.119 | 0.937 | 1.668 | 0.123 | 0.597 | 1.635 | 0.113 | 0.412 | 1.560 | 0.101 | 0.315 | 1.506 | 0.112 | 0.195 | 1.259 |
| 0.05 | 0.253 | 0.575 | 3.770 | 0.287 | 0.340 | 3.079 | 0.290 | 0.228 | 2.095 | 0.287 | 0.172 | 1.473 | 0.295 | 0.104 | 0.649 |
| 0.10 | 0.314 | 0.426 | 5.319 | 0.358 | 0.232 | 3.346 | 0.364 | 0.150 | 1.833 | 0.365 | 0.111 | 1.134 | 0.371 | 0.066 | 0.448 |
| 0.20 | 0.364 | 0.309 | 6.526 | 0.413 | 0.148 | 3.162 | 0.420 | 0.090 | 1.540 | 0.422 | 0.065 | 0.904 | 0.426 | 0.038 | 0.348 |
| 0.50 | 0.409 | 0.217 | 6.921 | 0.458 | 0.079 | 2.815 | 0.464 | 0.042 | 1.310 | 0.465 | 0.029 | 0.757 | 0.468 | 0.017 | 0.291 |

Table 59: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.01 | 0.214 | 0.943 | 1.590 | 0.232 | 0.602 | 1.462 | 0.225 | 0.416 | 1.270 | 0.214 | 0.318 | 1.119 | 0.225 | 0.197 | 0.712 |
| 0.05 | 0.490 | 0.608 | 2.436 | 0.567 | 0.359 | 1.352 | 0.581 | 0.243 | 0.719 | 0.582 | 0.182 | 0.443 | 0.594 | 0.110 | 0.175 |
| 0.10 | 0.615 | 0.478 | 2.449 | 0.712 | 0.256 | 1.062 | 0.731 | 0.165 | 0.507 | 0.736 | 0.120 | 0.296 | 0.746 | 0.071 | 0.114 |
| 0.20 | 0.718 | 0.383 | 2.243 | 0.824 | 0.174 | 0.857 | 0.843 | 0.102 | 0.395 | 0.848 | 0.072 | 0.227 | 0.855 | 0.042 | 0.087 |
| 0.50 | 0.809 | 0.327 | 1.954 | 0.916 | 0.107 | 0.716 | 0.930 | 0.049 | 0.329 | 0.933 | 0.033 | 0.189 | 0.937 | 0.019 | 0.073 |

Table 60: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.011 | 0.221 | 0.979 | −0.004 | 0.140 | 0.997 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 0.998 |
| 0.20 | 0.011 | 0.224 | 0.953 | −0.004 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.073 | 0.994 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.012 | 0.236 | 0.857 | −0.004 | 0.143 | 0.962 | 0.002 | 0.096 | 0.976 | −0.003 | 0.074 | 0.981 | 0.001 | 0.044 | 0.988 |

Table 61: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.995 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.010 | 0.224 | 0.955 | −0.005 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.990 | 0.001 | 0.044 | 0.997 |
| 0.50 | 0.007 | 0.236 | 0.859 | −0.006 | 0.143 | 0.958 | 0.001 | 0.096 | 0.974 | −0.004 | 0.074 | 0.968 | 0.001 | 0.044 | 0.987 |

Table 62: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.010 | 0.219 | 0.991 | −0.004 | 0.140 | 0.998 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.008 | 0.224 | 0.955 | −0.005 | 0.141 | 0.988 | 0.001 | 0.096 | 0.992 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.002 | 0.237 | 0.855 | −0.008 | 0.144 | 0.943 | 0.000 | 0.097 | 0.962 | −0.004 | 0.075 | 0.948 | 0.001 | 0.044 | 0.976 |

Table 63: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.009 | 0.219 | 0.992 | −0.005 | 0.140 | 0.997 | 0.001 | 0.095 | 0.998 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.008 | 0.221 | 0.980 | −0.005 | 0.141 | 0.992 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.997 |
| 0.20 | 0.004 | 0.225 | 0.947 | −0.007 | 0.142 | 0.972 | 0.001 | 0.096 | 0.978 | −0.004 | 0.075 | 0.960 | 0.001 | 0.044 | 0.983 |
| 0.50 | −0.014 | 0.244 | 0.802 | −0.014 | 0.151 | 0.855 | −0.001 | 0.102 | 0.879 | −0.005 | 0.079 | 0.852 | 0.000 | 0.047 | 0.889 |

Table 64: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.008 | 0.220 | 0.991 | −0.005 | 0.141 | 0.993 | 0.001 | 0.095 | 0.995 | −0.003 | 0.074 | 0.985 | 0.001 | 0.044 | 0.998 |
| 0.10 | 0.004 | 0.222 | 0.972 | −0.007 | 0.142 | 0.977 | 0.001 | 0.096 | 0.980 | −0.003 | 0.075 | 0.963 | 0.001 | 0.044 | 0.985 |
| 0.20 | −0.004 | 0.229 | 0.909 | −0.010 | 0.146 | 0.917 | 0.000 | 0.099 | 0.926 | −0.004 | 0.077 | 0.899 | 0.001 | 0.046 | 0.932 |
| 0.50 | −0.038 | 0.269 | 0.650 | −0.024 | 0.172 | 0.649 | −0.005 | 0.116 | 0.671 | −0.007 | 0.090 | 0.653 | −0.001 | 0.054 | 0.668 |

Table 65: Max$\underset{\sim}{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.01 | −0.001 | 0.217 | 0.999 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.05 | −0.001 | 0.218 | 0.997 | −0.002 | 0.145 | 1.002 | 0.005 | 0.101 | 1.002 | −0.001 | 0.071 | 1.001 | 0.000 | 0.045 | 1.002 |
| 0.10 | −0.001 | 0.218 | 0.991 | −0.002 | 0.144 | 1.003 | 0.005 | 0.101 | 1.003 | −0.001 | 0.071 | 1.003 | 0.000 | 0.045 | 1.005 |
| 0.20 | −0.001 | 0.220 | 0.975 | −0.002 | 0.144 | 1.004 | 0.005 | 0.101 | 1.004 | −0.001 | 0.071 | 1.005 | 0.000 | 0.045 | 1.009 |
| 0.50 | −0.002 | 0.230 | 0.893 | −0.002 | 0.145 | 0.990 | 0.005 | 0.101 | 0.997 | −0.001 | 0.071 | 1.007 | 0.000 | 0.044 | 1.021 |

Table 66: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.01 | −0.001 | 0.217 | 0.999 | −0.002 | 0.145 | 1.001 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.001 |
| 0.05 | −0.001 | 0.218 | 0.996 | −0.002 | 0.145 | 1.002 | 0.005 | 0.101 | 1.002 | −0.001 | 0.071 | 1.001 | 0.000 | 0.045 | 1.003 |
| 0.10 | −0.001 | 0.219 | 0.990 | −0.002 | 0.144 | 1.004 | 0.005 | 0.101 | 1.003 | −0.001 | 0.071 | 1.002 | 0.000 | 0.045 | 1.006 |
| 0.20 | −0.002 | 0.221 | 0.972 | −0.002 | 0.144 | 1.005 | 0.005 | 0.101 | 1.004 | −0.000 | 0.071 | 1.003 | 0.001 | 0.045 | 1.011 |
| 0.50 | −0.007 | 0.231 | 0.885 | −0.002 | 0.145 | 0.990 | 0.005 | 0.101 | 0.997 | −0.000 | 0.071 | 0.998 | 0.001 | 0.044 | 1.021 |

Table 67: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.01 | −0.001 | 0.217 | 0.999 | −0.002 | 0.145 | 1.001 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.001 |
| 0.05 | −0.001 | 0.218 | 0.995 | −0.002 | 0.145 | 1.003 | 0.005 | 0.101 | 1.002 | −0.001 | 0.071 | 1.001 | 0.000 | 0.045 | 1.003 |
| 0.10 | −0.002 | 0.219 | 0.988 | −0.002 | 0.144 | 1.004 | 0.005 | 0.101 | 1.003 | −0.000 | 0.071 | 1.001 | 0.001 | 0.045 | 1.006 |
| 0.20 | −0.004 | 0.221 | 0.967 | −0.002 | 0.144 | 1.004 | 0.006 | 0.101 | 1.002 | −0.000 | 0.071 | 0.999 | 0.001 | 0.045 | 1.010 |
| 0.50 | −0.011 | 0.233 | 0.869 | −0.003 | 0.146 | 0.979 | 0.006 | 0.102 | 0.986 | 0.000 | 0.072 | 0.979 | 0.002 | 0.045 | 1.008 |

Table 68: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.01 | −0.001 | 0.218 | 0.999 | −0.002 | 0.145 | 1.001 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.001 |
| 0.05 | −0.002 | 0.218 | 0.992 | −0.002 | 0.144 | 1.003 | 0.005 | 0.101 | 1.001 | −0.000 | 0.071 | 0.999 | 0.001 | 0.045 | 1.003 |
| 0.10 | −0.003 | 0.220 | 0.979 | −0.002 | 0.145 | 1.002 | 0.006 | 0.101 | 0.999 | 0.000 | 0.071 | 0.995 | 0.001 | 0.045 | 1.004 |
| 0.20 | −0.007 | 0.224 | 0.943 | −0.002 | 0.145 | 0.991 | 0.006 | 0.102 | 0.985 | 0.001 | 0.072 | 0.977 | 0.002 | 0.045 | 0.995 |
| 0.50 | −0.025 | 0.245 | 0.781 | −0.005 | 0.153 | 0.893 | 0.007 | 0.107 | 0.892 | 0.002 | 0.076 | 0.868 | 0.006 | 0.047 | 0.899 |

Table 69: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.01 | −0.001 | 0.218 | 0.998 | −0.002 | 0.145 | 1.001 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.001 |
| 0.05 | −0.003 | 0.219 | 0.984 | −0.002 | 0.145 | 1.001 | 0.006 | 0.101 | 0.997 | 0.000 | 0.071 | 0.994 | 0.001 | 0.045 | 1.001 |
| 0.10 | −0.006 | 0.222 | 0.956 | −0.001 | 0.145 | 0.990 | 0.007 | 0.102 | 0.981 | 0.001 | 0.072 | 0.975 | 0.002 | 0.045 | 0.989 |
| 0.20 | −0.014 | 0.232 | 0.877 | −0.002 | 0.150 | 0.936 | 0.008 | 0.105 | 0.919 | 0.003 | 0.074 | 0.907 | 0.005 | 0.046 | 0.928 |
| 0.50 | −0.046 | 0.278 | 0.597 | −0.008 | 0.176 | 0.673 | 0.009 | 0.126 | 0.642 | 0.006 | 0.091 | 0.604 | 0.013 | 0.059 | 0.561 |

Table 70: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.01 | −0.010 | 0.234 | 1.002 | −0.002 | 0.148 | 1.002 | 0.001 | 0.103 | 1.003 | 0.002 | 0.074 | 1.002 | −0.001 | 0.046 | 1.002 |
| 0.05 | −0.010 | 0.234 | 1.007 | −0.002 | 0.147 | 1.009 | 0.001 | 0.103 | 1.012 | 0.002 | 0.074 | 1.012 | −0.001 | 0.045 | 1.011 |
| 0.10 | −0.010 | 0.233 | 1.011 | −0.002 | 0.147 | 1.016 | 0.001 | 0.102 | 1.024 | 0.002 | 0.073 | 1.023 | −0.001 | 0.045 | 1.022 |
| 0.20 | −0.011 | 0.233 | 1.010 | −0.002 | 0.146 | 1.028 | 0.001 | 0.101 | 1.046 | 0.002 | 0.072 | 1.046 | −0.001 | 0.045 | 1.044 |
| 0.50 | −0.013 | 0.239 | 0.962 | −0.002 | 0.145 | 1.035 | 0.001 | 0.099 | 1.097 | 0.002 | 0.070 | 1.109 | −0.001 | 0.043 | 1.108 |

Table 71: $\text{Max}\underset{\sim}{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.01 | −0.010 | 0.234 | 1.002 | −0.002 | 0.148 | 1.002 | 0.001 | 0.103 | 1.003 | 0.002 | 0.074 | 1.002 | −0.000 | 0.046 | 1.002 |
| 0.05 | −0.010 | 0.234 | 1.006 | −0.002 | 0.147 | 1.009 | 0.002 | 0.103 | 1.012 | 0.002 | 0.073 | 1.012 | 0.000 | 0.045 | 1.011 |
| 0.10 | −0.010 | 0.234 | 1.008 | −0.001 | 0.147 | 1.017 | 0.002 | 0.102 | 1.024 | 0.003 | 0.073 | 1.023 | 0.001 | 0.045 | 1.022 |
| 0.20 | −0.011 | 0.234 | 1.005 | −0.001 | 0.146 | 1.028 | 0.003 | 0.101 | 1.044 | 0.004 | 0.072 | 1.043 | 0.002 | 0.045 | 1.041 |
| 0.50 | −0.015 | 0.241 | 0.947 | 0.001 | 0.145 | 1.033 | 0.006 | 0.099 | 1.087 | 0.007 | 0.070 | 1.092 | 0.005 | 0.044 | 1.083 |

Table 72: $\text{Max}\underset{\sim}{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.01 | −0.010 | 0.234 | 1.001 | −0.002 | 0.148 | 1.002 | 0.002 | 0.103 | 1.003 | 0.002 | 0.074 | 1.002 | −0.000 | 0.046 | 1.002 |
| 0.05 | −0.010 | 0.234 | 1.005 | −0.001 | 0.147 | 1.009 | 0.002 | 0.103 | 1.012 | 0.003 | 0.073 | 1.011 | 0.001 | 0.045 | 1.011 |
| 0.10 | −0.010 | 0.234 | 1.006 | −0.000 | 0.147 | 1.016 | 0.003 | 0.102 | 1.022 | 0.004 | 0.073 | 1.021 | 0.002 | 0.045 | 1.019 |
| 0.20 | −0.011 | 0.235 | 0.998 | 0.001 | 0.146 | 1.025 | 0.005 | 0.102 | 1.038 | 0.006 | 0.072 | 1.035 | 0.004 | 0.045 | 1.029 |
| 0.50 | −0.017 | 0.244 | 0.924 | 0.004 | 0.147 | 1.015 | 0.010 | 0.100 | 1.054 | 0.011 | 0.071 | 1.049 | 0.010 | 0.044 | 1.012 |

Table 73: $\text{Max}\underset{\sim}{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.01 | −0.010 | 0.235 | 1.001 | −0.002 | 0.148 | 1.002 | 0.002 | 0.103 | 1.002 | 0.002 | 0.074 | 1.002 | 0.000 | 0.046 | 1.002 |
| 0.05 | −0.010 | 0.235 | 1.000 | 0.000 | 0.147 | 1.008 | 0.004 | 0.103 | 1.009 | 0.005 | 0.074 | 1.007 | 0.002 | 0.045 | 1.006 |
| 0.10 | −0.010 | 0.235 | 0.994 | 0.002 | 0.147 | 1.009 | 0.007 | 0.103 | 1.011 | 0.007 | 0.073 | 1.006 | 0.005 | 0.045 | 0.997 |
| 0.20 | −0.012 | 0.239 | 0.966 | 0.006 | 0.148 | 0.997 | 0.012 | 0.103 | 0.996 | 0.012 | 0.074 | 0.983 | 0.010 | 0.046 | 0.947 |
| 0.50 | −0.024 | 0.259 | 0.818 | 0.014 | 0.157 | 0.880 | 0.024 | 0.109 | 0.860 | 0.026 | 0.078 | 0.806 | 0.026 | 0.050 | 0.659 |

Table 74: $\text{Max}\underset{\sim}{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.01 | −0.010 | 0.235 | 0.999 | −0.001 | 0.148 | 1.002 | 0.002 | 0.103 | 1.002 | 0.003 | 0.074 | 1.002 | 0.001 | 0.046 | 1.002 |
| 0.05 | −0.009 | 0.236 | 0.989 | 0.002 | 0.148 | 1.001 | 0.007 | 0.103 | 0.999 | 0.007 | 0.074 | 0.995 | 0.005 | 0.046 | 0.987 |
| 0.10 | −0.010 | 0.239 | 0.964 | 0.007 | 0.149 | 0.980 | 0.012 | 0.104 | 0.972 | 0.013 | 0.074 | 0.959 | 0.011 | 0.046 | 0.924 |
| 0.20 | −0.013 | 0.249 | 0.886 | 0.015 | 0.156 | 0.893 | 0.022 | 0.109 | 0.862 | 0.023 | 0.078 | 0.821 | 0.022 | 0.049 | 0.714 |
| 0.50 | −0.035 | 0.297 | 0.616 | 0.031 | 0.194 | 0.566 | 0.048 | 0.141 | 0.483 | 0.054 | 0.106 | 0.386 | 0.062 | 0.075 | 0.222 |

Table 75: $\text{Max}\underset{\sim}{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.01 | 0.005 | 0.239 | 1.005 | −0.003 | 0.161 | 1.005 | 0.004 | 0.112 | 1.005 | −0.000 | 0.078 | 1.004 | −0.001 | 0.050 | 1.005 |
| 0.05 | 0.005 | 0.237 | 1.023 | −0.003 | 0.159 | 1.024 | 0.003 | 0.110 | 1.025 | −0.000 | 0.078 | 1.023 | −0.001 | 0.049 | 1.023 |
| 0.10 | 0.004 | 0.235 | 1.040 | −0.003 | 0.157 | 1.047 | 0.003 | 0.109 | 1.048 | −0.000 | 0.077 | 1.045 | −0.001 | 0.049 | 1.047 |
| 0.20 | 0.004 | 0.233 | 1.063 | −0.003 | 0.155 | 1.086 | 0.003 | 0.107 | 1.093 | −0.000 | 0.075 | 1.088 | −0.001 | 0.048 | 1.093 |
| 0.50 | 0.003 | 0.233 | 1.065 | −0.003 | 0.149 | 1.169 | 0.003 | 0.102 | 1.211 | −0.000 | 0.071 | 1.214 | −0.001 | 0.045 | 1.234 |

Table 76: Max$\underset{\sim}{D}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.01 | 0.005 | 0.239 | 1.005 | −0.003 | 0.161 | 1.005 | 0.004 | 0.112 | 1.005 | 0.000 | 0.078 | 1.004 | −0.001 | 0.050 | 1.005 |
| 0.05 | 0.005 | 0.237 | 1.021 | −0.002 | 0.159 | 1.024 | 0.005 | 0.110 | 1.023 | 0.001 | 0.078 | 1.022 | 0.000 | 0.049 | 1.023 |
| 0.10 | 0.006 | 0.236 | 1.037 | −0.002 | 0.157 | 1.046 | 0.006 | 0.109 | 1.046 | 0.002 | 0.077 | 1.043 | 0.001 | 0.049 | 1.044 |
| 0.20 | 0.006 | 0.233 | 1.058 | 0.000 | 0.155 | 1.084 | 0.007 | 0.107 | 1.086 | 0.004 | 0.075 | 1.082 | 0.003 | 0.048 | 1.083 |
| 0.50 | 0.005 | 0.234 | 1.050 | 0.004 | 0.150 | 1.157 | 0.012 | 0.102 | 1.186 | 0.009 | 0.072 | 1.180 | 0.009 | 0.045 | 1.164 |

Table 77: Max$\underset{\sim}{D}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.01 | 0.005 | 0.239 | 1.004 | −0.003 | 0.161 | 1.005 | 0.004 | 0.112 | 1.005 | 0.000 | 0.078 | 1.004 | −0.000 | 0.050 | 1.005 |
| 0.05 | 0.006 | 0.238 | 1.019 | −0.001 | 0.159 | 1.024 | 0.006 | 0.111 | 1.022 | 0.002 | 0.078 | 1.020 | 0.001 | 0.049 | 1.021 |
| 0.10 | 0.007 | 0.236 | 1.033 | 0.000 | 0.158 | 1.044 | 0.008 | 0.109 | 1.041 | 0.004 | 0.077 | 1.038 | 0.004 | 0.049 | 1.037 |
| 0.20 | 0.008 | 0.234 | 1.048 | 0.004 | 0.155 | 1.077 | 0.011 | 0.107 | 1.073 | 0.008 | 0.076 | 1.065 | 0.008 | 0.048 | 1.051 |
| 0.50 | 0.008 | 0.237 | 1.021 | 0.011 | 0.152 | 1.122 | 0.021 | 0.103 | 1.124 | 0.018 | 0.073 | 1.090 | 0.019 | 0.046 | 0.988 |

Table 78: Max$\underset{\sim}{D}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.01 | 0.006 | 0.240 | 1.003 | −0.002 | 0.161 | 1.005 | 0.005 | 0.112 | 1.004 | 0.001 | 0.078 | 1.004 | 0.000 | 0.050 | 1.004 |
| 0.05 | 0.009 | 0.238 | 1.012 | 0.001 | 0.159 | 1.021 | 0.009 | 0.111 | 1.014 | 0.005 | 0.078 | 1.012 | 0.005 | 0.049 | 1.007 |
| 0.10 | 0.011 | 0.238 | 1.015 | 0.006 | 0.158 | 1.032 | 0.014 | 0.110 | 1.017 | 0.010 | 0.078 | 1.008 | 0.010 | 0.049 | 0.981 |
| 0.20 | 0.015 | 0.239 | 1.001 | 0.014 | 0.158 | 1.030 | 0.023 | 0.110 | 0.994 | 0.020 | 0.078 | 0.960 | 0.021 | 0.049 | 0.860 |
| 0.50 | 0.017 | 0.256 | 0.875 | 0.032 | 0.165 | 0.924 | 0.048 | 0.114 | 0.817 | 0.047 | 0.082 | 0.693 | 0.051 | 0.054 | 0.449 |

Table 79: Max$\underset{\sim}{D}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.01 | 0.007 | 0.240 | 1.001 | −0.001 | 0.161 | 1.005 | 0.006 | 0.112 | 1.002 | 0.002 | 0.078 | 1.002 | 0.001 | 0.050 | 1.003 |
| 0.05 | 0.013 | 0.240 | 0.994 | 0.006 | 0.160 | 1.009 | 0.014 | 0.111 | 0.992 | 0.011 | 0.078 | 0.985 | 0.010 | 0.050 | 0.959 |
| 0.10 | 0.019 | 0.243 | 0.966 | 0.015 | 0.161 | 0.987 | 0.025 | 0.113 | 0.944 | 0.021 | 0.079 | 0.914 | 0.022 | 0.051 | 0.818 |
| 0.20 | 0.027 | 0.255 | 0.878 | 0.032 | 0.168 | 0.887 | 0.044 | 0.118 | 0.783 | 0.042 | 0.084 | 0.697 | 0.044 | 0.055 | 0.497 |
| 0.50 | 0.030 | 0.307 | 0.604 | 0.067 | 0.206 | 0.552 | 0.097 | 0.153 | 0.380 | 0.101 | 0.117 | 0.259 | 0.116 | 0.086 | 0.118 |

Table 80: Max$\underset{\sim}{D}_{\alpha}$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.01 | 0.004 | 0.267 | 1.011 | −0.004 | 0.177 | 1.010 | −0.004 | 0.123 | 1.010 | −0.002 | 0.084 | 1.010 | 0.002 | 0.054 | 1.009 |
| 0.05 | 0.004 | 0.262 | 1.051 | −0.004 | 0.174 | 1.047 | −0.003 | 0.121 | 1.048 | −0.002 | 0.082 | 1.048 | 0.002 | 0.053 | 1.047 |
| 0.10 | 0.004 | 0.257 | 1.095 | −0.004 | 0.170 | 1.091 | −0.003 | 0.118 | 1.095 | −0.002 | 0.080 | 1.096 | 0.002 | 0.052 | 1.096 |
| 0.20 | 0.004 | 0.249 | 1.166 | −0.004 | 0.164 | 1.172 | −0.003 | 0.114 | 1.188 | −0.002 | 0.077 | 1.192 | 0.002 | 0.049 | 1.194 |
| 0.50 | 0.004 | 0.238 | 1.276 | −0.003 | 0.152 | 1.379 | −0.002 | 0.103 | 1.451 | −0.002 | 0.069 | 1.489 | 0.001 | 0.044 | 1.506 |

Table 81: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.01 | 0.004 | 0.267 | 1.011 | −0.004 | 0.177 | 1.010 | −0.003 | 0.123 | 1.010 | −0.002 | 0.084 | 1.010 | 0.002 | 0.054 | 1.009 |
| 0.05 | 0.006 | 0.262 | 1.051 | −0.002 | 0.174 | 1.048 | −0.001 | 0.121 | 1.050 | −0.000 | 0.082 | 1.049 | 0.004 | 0.053 | 1.042 |
| 0.10 | 0.007 | 0.257 | 1.094 | −0.000 | 0.170 | 1.092 | 0.001 | 0.118 | 1.098 | 0.002 | 0.080 | 1.097 | 0.006 | 0.052 | 1.080 |
| 0.20 | 0.009 | 0.249 | 1.163 | 0.003 | 0.164 | 1.173 | 0.005 | 0.114 | 1.187 | 0.006 | 0.077 | 1.186 | 0.010 | 0.050 | 1.144 |
| 0.50 | 0.013 | 0.238 | 1.267 | 0.011 | 0.152 | 1.365 | 0.014 | 0.103 | 1.419 | 0.016 | 0.069 | 1.403 | 0.020 | 0.044 | 1.235 |

Table 82: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.01 | 0.005 | 0.267 | 1.011 | −0.003 | 0.177 | 1.010 | −0.003 | 0.123 | 1.011 | −0.001 | 0.084 | 1.010 | 0.003 | 0.054 | 1.008 |
| 0.05 | 0.007 | 0.262 | 1.050 | −0.000 | 0.174 | 1.048 | 0.001 | 0.121 | 1.050 | 0.002 | 0.082 | 1.049 | 0.006 | 0.053 | 1.033 |
| 0.10 | 0.010 | 0.257 | 1.092 | 0.004 | 0.170 | 1.091 | 0.005 | 0.118 | 1.095 | 0.006 | 0.080 | 1.090 | 0.010 | 0.052 | 1.049 |
| 0.20 | 0.015 | 0.250 | 1.155 | 0.010 | 0.165 | 1.164 | 0.013 | 0.114 | 1.169 | 0.014 | 0.077 | 1.148 | 0.018 | 0.050 | 1.037 |
| 0.50 | 0.023 | 0.241 | 1.233 | 0.026 | 0.154 | 1.304 | 0.031 | 0.105 | 1.288 | 0.033 | 0.071 | 1.163 | 0.038 | 0.045 | 0.825 |

Table 83: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.01 | 0.006 | 0.267 | 1.011 | −0.002 | 0.177 | 1.011 | −0.001 | 0.123 | 1.011 | −0.000 | 0.084 | 1.011 | 0.004 | 0.054 | 1.004 |
| 0.05 | 0.013 | 0.263 | 1.045 | 0.006 | 0.174 | 1.045 | 0.008 | 0.121 | 1.045 | 0.009 | 0.082 | 1.036 | 0.013 | 0.053 | 0.982 |
| 0.10 | 0.020 | 0.259 | 1.073 | 0.015 | 0.171 | 1.073 | 0.018 | 0.119 | 1.060 | 0.019 | 0.081 | 1.026 | 0.024 | 0.052 | 0.888 |
| 0.20 | 0.031 | 0.255 | 1.097 | 0.031 | 0.168 | 1.087 | 0.037 | 0.117 | 1.023 | 0.038 | 0.079 | 0.911 | 0.044 | 0.052 | 0.638 |
| 0.50 | 0.050 | 0.260 | 1.027 | 0.069 | 0.168 | 0.961 | 0.082 | 0.118 | 0.742 | 0.088 | 0.080 | 0.503 | 0.097 | 0.053 | 0.241 |

Table 84: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.01 | 0.008 | 0.267 | 1.010 | −0.000 | 0.177 | 1.011 | 0.001 | 0.123 | 1.012 | 0.002 | 0.084 | 1.011 | 0.006 | 0.054 | 0.995 |
| 0.05 | 0.021 | 0.264 | 1.026 | 0.016 | 0.175 | 1.028 | 0.019 | 0.122 | 1.012 | 0.020 | 0.083 | 0.980 | 0.024 | 0.054 | 0.847 |
| 0.10 | 0.036 | 0.265 | 1.011 | 0.034 | 0.175 | 1.000 | 0.039 | 0.122 | 0.929 | 0.041 | 0.083 | 0.829 | 0.046 | 0.054 | 0.579 |
| 0.20 | 0.059 | 0.273 | 0.927 | 0.067 | 0.180 | 0.857 | 0.078 | 0.129 | 0.675 | 0.080 | 0.088 | 0.501 | 0.088 | 0.058 | 0.261 |
| 0.50 | 0.091 | 0.322 | 0.644 | 0.140 | 0.218 | 0.471 | 0.171 | 0.167 | 0.268 | 0.186 | 0.118 | 0.146 | 0.208 | 0.085 | 0.058 |

Table 85: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.01 | −0.008 | 0.275 | 1.015 | −0.000 | 0.191 | 1.015 | −0.001 | 0.133 | 1.015 | −0.001 | 0.092 | 1.014 | 0.000 | 0.056 | 1.014 |
| 0.05 | −0.008 | 0.267 | 1.072 | −0.000 | 0.185 | 1.075 | −0.001 | 0.130 | 1.073 | −0.001 | 0.089 | 1.071 | 0.000 | 0.055 | 1.072 |
| 0.10 | −0.007 | 0.260 | 1.135 | −0.000 | 0.180 | 1.147 | −0.001 | 0.125 | 1.146 | −0.001 | 0.086 | 1.144 | 0.000 | 0.053 | 1.146 |
| 0.20 | −0.007 | 0.248 | 1.245 | −0.000 | 0.170 | 1.284 | −0.001 | 0.118 | 1.294 | −0.001 | 0.081 | 1.292 | 0.000 | 0.050 | 1.299 |
| 0.50 | −0.007 | 0.228 | 1.473 | −0.000 | 0.150 | 1.653 | −0.001 | 0.102 | 1.744 | −0.000 | 0.070 | 1.765 | 0.000 | 0.042 | 1.806 |

Table 86: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.01 | −0.007 | 0.275 | 1.015 | 0.000 | 0.191 | 1.015 | −0.000 | 0.133 | 1.015 | −0.000 | 0.092 | 1.014 | 0.001 | 0.056 | 1.014 |
| 0.05 | −0.005 | 0.267 | 1.072 | 0.003 | 0.185 | 1.075 | 0.002 | 0.130 | 1.072 | 0.003 | 0.089 | 1.071 | 0.004 | 0.055 | 1.066 |
| 0.10 | −0.003 | 0.260 | 1.136 | 0.006 | 0.180 | 1.146 | 0.005 | 0.126 | 1.143 | 0.006 | 0.086 | 1.140 | 0.007 | 0.053 | 1.124 |
| 0.20 | 0.001 | 0.248 | 1.243 | 0.010 | 0.170 | 1.277 | 0.011 | 0.118 | 1.279 | 0.011 | 0.081 | 1.267 | 0.012 | 0.050 | 1.217 |
| 0.50 | 0.008 | 0.230 | 1.454 | 0.021 | 0.150 | 1.605 | 0.023 | 0.102 | 1.641 | 0.024 | 0.070 | 1.562 | 0.025 | 0.043 | 1.305 |

Table 87: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.01 | −0.007 | 0.275 | 1.016 | 0.001 | 0.191 | 1.015 | 0.000 | 0.133 | 1.014 | 0.001 | 0.092 | 1.015 | 0.002 | 0.056 | 1.013 |
| 0.05 | −0.003 | 0.267 | 1.072 | 0.006 | 0.185 | 1.074 | 0.006 | 0.130 | 1.069 | 0.006 | 0.089 | 1.068 | 0.007 | 0.055 | 1.052 |
| 0.10 | 0.002 | 0.260 | 1.134 | 0.011 | 0.180 | 1.140 | 0.011 | 0.126 | 1.132 | 0.012 | 0.086 | 1.122 | 0.013 | 0.053 | 1.072 |
| 0.20 | 0.010 | 0.249 | 1.233 | 0.021 | 0.170 | 1.254 | 0.022 | 0.119 | 1.234 | 0.023 | 0.082 | 1.188 | 0.024 | 0.050 | 1.032 |
| 0.50 | 0.023 | 0.233 | 1.396 | 0.042 | 0.153 | 1.473 | 0.047 | 0.104 | 1.384 | 0.049 | 0.071 | 1.146 | 0.051 | 0.044 | 0.717 |

Table 88: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.01 | −0.005 | 0.275 | 1.016 | 0.003 | 0.191 | 1.015 | 0.002 | 0.133 | 1.014 | 0.003 | 0.092 | 1.014 | 0.004 | 0.057 | 1.009 |
| 0.05 | 0.005 | 0.268 | 1.068 | 0.015 | 0.186 | 1.064 | 0.015 | 0.130 | 1.050 | 0.016 | 0.089 | 1.037 | 0.017 | 0.055 | 0.968 |
| 0.10 | 0.016 | 0.262 | 1.113 | 0.029 | 0.181 | 1.100 | 0.030 | 0.127 | 1.059 | 0.031 | 0.087 | 1.000 | 0.032 | 0.054 | 0.813 |
| 0.20 | 0.034 | 0.255 | 1.156 | 0.053 | 0.175 | 1.107 | 0.057 | 0.123 | 0.987 | 0.059 | 0.084 | 0.812 | 0.061 | 0.053 | 0.501 |
| 0.50 | 0.067 | 0.256 | 1.093 | 0.106 | 0.170 | 0.927 | 0.119 | 0.118 | 0.646 | 0.124 | 0.081 | 0.387 | 0.129 | 0.051 | 0.167 |

Table 89: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.01 | −0.002 | 0.275 | 1.016 | 0.006 | 0.191 | 1.014 | 0.006 | 0.134 | 1.011 | 0.006 | 0.092 | 1.011 | 0.007 | 0.057 | 0.995 |
| 0.05 | 0.018 | 0.270 | 1.048 | 0.030 | 0.187 | 1.027 | 0.032 | 0.132 | 0.987 | 0.032 | 0.090 | 0.934 | 0.034 | 0.056 | 0.758 |
| 0.10 | 0.040 | 0.269 | 1.038 | 0.058 | 0.186 | 0.970 | 0.062 | 0.131 | 0.855 | 0.063 | 0.090 | 0.708 | 0.065 | 0.056 | 0.436 |
| 0.20 | 0.076 | 0.276 | 0.937 | 0.107 | 0.192 | 0.768 | 0.116 | 0.137 | 0.560 | 0.120 | 0.094 | 0.367 | 0.124 | 0.060 | 0.171 |
| 0.50 | 0.133 | 0.324 | 0.625 | 0.211 | 0.226 | 0.387 | 0.243 | 0.166 | 0.208 | 0.259 | 0.120 | 0.105 | 0.274 | 0.081 | 0.040 |

Table 90: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.011 | 0.221 | 0.979 | −0.004 | 0.140 | 0.997 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 0.998 |
| 0.20 | 0.011 | 0.224 | 0.953 | −0.004 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.073 | 0.994 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.012 | 0.236 | 0.857 | −0.004 | 0.143 | 0.962 | 0.002 | 0.096 | 0.976 | −0.003 | 0.074 | 0.981 | 0.001 | 0.044 | 0.988 |

Table 91: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.220 | 0.990 | −0.004 | 0.140 | 0.999 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 0.999 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.995 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.010 | 0.224 | 0.955 | −0.005 | 0.141 | 0.991 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.990 | 0.001 | 0.044 | 0.997 |
| 0.50 | 0.007 | 0.236 | 0.859 | −0.006 | 0.143 | 0.958 | 0.001 | 0.096 | 0.974 | −0.004 | 0.074 | 0.968 | 0.001 | 0.044 | 0.987 |

Table 92: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.219 | 0.998 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.010 | 0.219 | 0.991 | −0.004 | 0.140 | 0.998 | 0.001 | 0.095 | 0.999 | −0.003 | 0.073 | 0.997 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.010 | 0.221 | 0.980 | −0.005 | 0.140 | 0.996 | 0.001 | 0.095 | 0.997 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 0.999 |
| 0.20 | 0.008 | 0.224 | 0.955 | −0.005 | 0.141 | 0.988 | 0.001 | 0.096 | 0.992 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.996 |
| 0.50 | 0.002 | 0.237 | 0.855 | −0.008 | 0.144 | 0.943 | 0.000 | 0.097 | 0.962 | −0.004 | 0.075 | 0.948 | 0.001 | 0.044 | 0.976 |

Table 93: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.999 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.009 | 0.219 | 0.992 | −0.005 | 0.140 | 0.997 | 0.001 | 0.095 | 0.998 | −0.003 | 0.073 | 0.993 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.008 | 0.221 | 0.980 | −0.005 | 0.141 | 0.992 | 0.001 | 0.095 | 0.994 | −0.003 | 0.074 | 0.984 | 0.001 | 0.044 | 0.997 |
| 0.20 | 0.004 | 0.225 | 0.947 | −0.007 | 0.142 | 0.972 | 0.001 | 0.096 | 0.978 | −0.004 | 0.075 | 0.960 | 0.001 | 0.044 | 0.983 |
| 0.50 | −0.014 | 0.244 | 0.802 | −0.014 | 0.151 | 0.855 | −0.001 | 0.102 | 0.879 | −0.005 | 0.079 | 0.852 | 0.000 | 0.047 | 0.889 |

Table 94: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.010 | 0.219 | 0.999 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 0.998 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.008 | 0.220 | 0.991 | −0.005 | 0.141 | 0.993 | 0.001 | 0.095 | 0.995 | −0.003 | 0.074 | 0.985 | 0.001 | 0.044 | 0.998 |
| 0.10 | 0.004 | 0.222 | 0.972 | −0.007 | 0.142 | 0.977 | 0.001 | 0.096 | 0.980 | −0.003 | 0.075 | 0.963 | 0.001 | 0.044 | 0.985 |
| 0.20 | −0.004 | 0.229 | 0.909 | −0.010 | 0.146 | 0.917 | 0.000 | 0.099 | 0.926 | −0.004 | 0.077 | 0.899 | 0.001 | 0.046 | 0.932 |
| 0.50 | −0.038 | 0.269 | 0.650 | −0.024 | 0.172 | 0.649 | −0.005 | 0.116 | 0.671 | −0.007 | 0.090 | 0.653 | −0.001 | 0.054 | 0.668 |

Table 95: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.01 | 0.000 | 0.239 | 2.369 | −0.001 | 0.171 | 4.502 | 0.006 | 0.123 | 22.44 | −0.003 | 0.089 | 21.37 | 0.001 | 0.068 | 5.596 |
| 0.05 | −0.000 | 0.224 | 2.705 | −0.002 | 0.155 | 5.524 | 0.005 | 0.110 | 28.21 | −0.003 | 0.078 | 27.43 | 0.001 | 0.054 | 8.958 |
| 0.10 | −0.001 | 0.222 | 2.766 | −0.001 | 0.150 | 5.858 | 0.005 | 0.106 | 30.03 | −0.002 | 0.075 | 29.82 | 0.001 | 0.050 | 10.34 |
| 0.20 | −0.001 | 0.222 | 2.766 | −0.001 | 0.147 | 6.114 | 0.004 | 0.104 | 31.59 | −0.002 | 0.073 | 31.97 | 0.000 | 0.047 | 11.66 |
| 0.50 | −0.002 | 0.230 | 2.561 | −0.000 | 0.145 | 6.257 | 0.005 | 0.101 | 33.00 | −0.002 | 0.070 | 34.31 | 0.000 | 0.044 | 13.37 |

Table 96: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.01 | 0.001 | 0.240 | 2.363 | −0.001 | 0.172 | 4.473 | 0.007 | 0.124 | 22.03 | −0.003 | 0.089 | 21.45 | 0.003 | 0.070 | 5.368 |
| 0.05 | −0.000 | 0.224 | 2.698 | −0.000 | 0.155 | 5.515 | 0.006 | 0.110 | 27.78 | −0.001 | 0.079 | 27.36 | 0.005 | 0.056 | 8.200 |
| 0.10 | −0.001 | 0.222 | 2.757 | 0.000 | 0.150 | 5.854 | 0.006 | 0.107 | 29.60 | 0.000 | 0.076 | 29.46 | 0.006 | 0.053 | 9.109 |
| 0.20 | −0.002 | 0.222 | 2.753 | 0.001 | 0.147 | 6.104 | 0.007 | 0.104 | 31.14 | 0.002 | 0.074 | 31.16 | 0.007 | 0.051 | 9.752 |
| 0.50 | −0.006 | 0.231 | 2.534 | 0.001 | 0.146 | 6.211 | 0.008 | 0.102 | 32.44 | 0.004 | 0.072 | 32.45 | 0.010 | 0.050 | 10.15 |

Table 97: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.01 | 0.001 | 0.240 | 2.357 | 0.000 | 0.173 | 4.436 | 0.008 | 0.125 | 21.53 | −0.002 | 0.089 | 21.28 | 0.005 | 0.072 | 4.974 |
| 0.05 | −0.000 | 0.225 | 2.689 | 0.001 | 0.155 | 5.478 | 0.008 | 0.112 | 27.04 | 0.001 | 0.080 | 26.41 | 0.008 | 0.062 | 6.769 |
| 0.10 | −0.001 | 0.222 | 2.744 | 0.001 | 0.151 | 5.802 | 0.008 | 0.109 | 28.68 | 0.003 | 0.078 | 27.72 | 0.011 | 0.061 | 6.942 |
| 0.20 | −0.003 | 0.223 | 2.730 | 0.002 | 0.148 | 6.018 | 0.009 | 0.106 | 29.88 | 0.006 | 0.077 | 28.28 | 0.014 | 0.061 | 6.716 |
| 0.50 | −0.010 | 0.234 | 2.478 | 0.002 | 0.148 | 6.009 | 0.011 | 0.105 | 30.24 | 0.009 | 0.078 | 27.52 | 0.021 | 0.063 | 5.923 |

Table 98: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.01 | 0.001 | 0.241 | 2.330 | 0.002 | 0.176 | 4.286 | 0.010 | 0.131 | 19.63 | 0.001 | 0.093 | 19.56 | 0.010 | 0.087 | 3.456 |
| 0.05 | 0.000 | 0.227 | 2.640 | 0.004 | 0.160 | 5.201 | 0.012 | 0.120 | 23.50 | 0.007 | 0.090 | 20.80 | 0.019 | 0.090 | 3.125 |
| 0.10 | −0.001 | 0.225 | 2.674 | 0.006 | 0.156 | 5.403 | 0.014 | 0.118 | 23.95 | 0.011 | 0.092 | 19.51 | 0.026 | 0.097 | 2.591 |
| 0.20 | −0.005 | 0.228 | 2.612 | 0.007 | 0.156 | 5.413 | 0.017 | 0.120 | 23.25 | 0.016 | 0.098 | 17.31 | 0.036 | 0.108 | 2.015 |
| 0.50 | −0.022 | 0.248 | 2.186 | 0.006 | 0.165 | 4.842 | 0.022 | 0.129 | 19.87 | 0.025 | 0.111 | 13.15 | 0.057 | 0.128 | 1.345 |

Table 99: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.01 | 0.002 | 0.245 | 2.264 | 0.004 | 0.184 | 3.918 | 0.013 | 0.146 | 15.81 | 0.005 | 0.106 | 14.91 | 0.019 | 0.123 | 1.692 |
| 0.05 | 0.001 | 0.233 | 2.505 | 0.009 | 0.174 | 4.385 | 0.019 | 0.143 | 16.40 | 0.016 | 0.119 | 11.66 | 0.040 | 0.156 | 1.011 |
| 0.10 | −0.001 | 0.234 | 2.480 | 0.012 | 0.175 | 4.304 | 0.024 | 0.148 | 15.04 | 0.024 | 0.134 | 9.123 | 0.057 | 0.182 | 0.721 |
| 0.20 | −0.008 | 0.243 | 2.295 | 0.015 | 0.183 | 3.932 | 0.031 | 0.162 | 12.52 | 0.035 | 0.155 | 6.704 | 0.082 | 0.214 | 0.498 |
| 0.50 | −0.041 | 0.288 | 1.599 | 0.015 | 0.217 | 2.794 | 0.041 | 0.198 | 8.282 | 0.058 | 0.197 | 4.004 | 0.134 | 0.267 | 0.294 |

Table 100: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.689 | 16.848 | 1.000 |
| 0.01 | −0.019 | 0.308 | 72.40 | 0.008 | 0.254 | 8.362 | 0.001 | 0.192 | 94.37 | −0.000 | 0.151 | 82.27 | 0.000 | 0.117 | $2.08 \cdot 10^4$ |
| 0.05 | −0.014 | 0.262 | 99.63 | 0.004 | 0.188 | 15.27 | 0.002 | 0.141 | 174.1 | 0.001 | 0.108 | 160.6 | −0.000 | 0.077 | $4.8 \cdot 10^4$ |
| 0.10 | −0.013 | 0.249 | 110.4 | 0.002 | 0.169 | 18.94 | 0.001 | 0.125 | 221.1 | 0.001 | 0.094 | 210.5 | −0.000 | 0.064 | $6.86 \cdot 10^4$ |
| 0.20 | −0.013 | 0.240 | 118.9 | 0.000 | 0.155 | 22.55 | 0.001 | 0.112 | 275.4 | 0.001 | 0.082 | 275.1 | −0.000 | 0.054 | $9.83 \cdot 10^4$ |
| 0.50 | −0.014 | 0.239 | 120.3 | −0.001 | 0.144 | 26.08 | 0.001 | 0.099 | 355.1 | 0.001 | 0.069 | 393.9 | −0.000 | 0.042 | $1.61 \cdot 10^5$ |

Table 101: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.689 | 16.848 | 1.000 |
| 0.01 | −0.018 | 0.307 | 72.97 | 0.011 | 0.255 | 8.295 | 0.005 | 0.192 | 94.24 | 0.005 | 0.151 | 81.67 | 0.009 | 0.119 | $1.99 \cdot 10^4$ |
| 0.05 | −0.012 | 0.262 | 100.0 | 0.010 | 0.190 | 14.88 | 0.010 | 0.143 | 170.0 | 0.011 | 0.110 | 153.4 | 0.017 | 0.082 | $4.1 \cdot 10^4$ |
| 0.10 | −0.010 | 0.249 | 110.4 | 0.010 | 0.172 | 18.27 | 0.012 | 0.128 | 211.9 | 0.015 | 0.097 | 193.1 | 0.022 | 0.071 | $5.17 \cdot 10^4$ |
| 0.20 | −0.009 | 0.241 | 118.2 | 0.011 | 0.158 | 21.54 | 0.015 | 0.115 | 256.6 | 0.020 | 0.087 | 235.8 | 0.029 | 0.062 | $6.02 \cdot 10^4$ |
| 0.50 | −0.012 | 0.241 | 117.8 | 0.013 | 0.148 | 24.61 | 0.021 | 0.104 | 311.6 | 0.028 | 0.076 | 289.0 | 0.040 | 0.054 | $6.36 \cdot 10^4$ |

Table 102: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.689 | 16.848 | 1.000 |
| 0.01 | −0.017 | 0.306 | 73.44 | 0.014 | 0.256 | 8.193 | 0.009 | 0.193 | 93.09 | 0.010 | 0.153 | 79.25 | 0.017 | 0.124 | $1.8 \cdot 10^4$ |
| 0.05 | −0.010 | 0.262 | 100.00 | 0.016 | 0.194 | 14.22 | 0.017 | 0.147 | 159.7 | 0.021 | 0.116 | 135.0 | 0.034 | 0.094 | $2.88 \cdot 10^4$ |
| 0.10 | −0.007 | 0.250 | 109.7 | 0.018 | 0.177 | 17.08 | 0.023 | 0.133 | 190.2 | 0.029 | 0.106 | 155.2 | 0.044 | 0.087 | $3 \cdot 10^4$ |
| 0.20 | −0.006 | 0.243 | 116.3 | 0.021 | 0.165 | 19.55 | 0.029 | 0.124 | 215.4 | 0.038 | 0.099 | 166.6 | 0.057 | 0.083 | $2.81 \cdot 10^4$ |
| 0.50 | −0.011 | 0.247 | 112.9 | 0.026 | 0.157 | 21.30 | 0.040 | 0.116 | 229.9 | 0.054 | 0.093 | 162.4 | 0.079 | 0.079 | $2.28 \cdot 10^4$ |

Table 103: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.689 | 16.848 | 1.000 |
| 0.01 | −0.013 | 0.304 | 74.26 | 0.023 | 0.264 | 7.704 | 0.020 | 0.202 | 84.94 | 0.025 | 0.168 | 65.04 | 0.042 | 0.154 | $1.11 \cdot 10^4$ |
| 0.05 | −0.003 | 0.266 | 97.57 | 0.035 | 0.215 | 11.43 | 0.040 | 0.170 | 113.8 | 0.051 | 0.150 | 74.25 | 0.083 | 0.153 | $9.42 \cdot 10^3$ |
| 0.10 | 0.001 | 0.257 | 103.8 | 0.043 | 0.205 | 12.31 | 0.054 | 0.167 | 113.1 | 0.070 | 0.153 | 66.50 | 0.111 | 0.160 | $7.55 \cdot 10^3$ |
| 0.20 | 0.002 | 0.256 | 105.0 | 0.052 | 0.202 | 12.43 | 0.071 | 0.169 | 103.4 | 0.094 | 0.159 | 54.86 | 0.147 | 0.168 | $5.72 \cdot 10^3$ |
| 0.50 | −0.008 | 0.274 | 91.26 | 0.065 | 0.209 | 11.26 | 0.098 | 0.182 | 81.67 | 0.136 | 0.172 | 38.68 | 0.210 | 0.175 | $3.81 \cdot 10^3$ |

Table 104: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.689 | 16.848 | 1.000 |
| 0.01 | −0.008 | 0.305 | 73.74 | 0.037 | 0.285 | 6.521 | 0.037 | 0.229 | 64.86 | 0.048 | 0.211 | 40.00 | 0.083 | 0.230 | $4.75 \cdot 10^3$ |
| 0.05 | 0.007 | 0.279 | 88.06 | 0.066 | 0.269 | 7.028 | 0.078 | 0.235 | 56.88 | 0.102 | 0.236 | 28.23 | 0.172 | 0.281 | $2.62 \cdot 10^3$ |
| 0.10 | 0.013 | 0.281 | 87.24 | 0.083 | 0.279 | 6.386 | 0.106 | 0.254 | 46.03 | 0.142 | 0.263 | 20.99 | 0.236 | 0.310 | $1.87 \cdot 10^3$ |
| 0.20 | 0.015 | 0.293 | 79.80 | 0.104 | 0.296 | 5.485 | 0.143 | 0.281 | 34.99 | 0.197 | 0.294 | 14.92 | 0.323 | 0.336 | $1.31 \cdot 10^3$ |
| 0.50 | −0.005 | 0.346 | 57.45 | 0.131 | 0.337 | 4.135 | 0.203 | 0.332 | 23.06 | 0.296 | 0.341 | 9.176 | 0.471 | 0.350 | 825.7 |

Table 105: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.099 | 6.404 | 1.000 | −0.272 | 4.787 | 1.000 | 0.194 | 6.527 | 1.000 | −0.009 | 1.426 | 1.000 | −0.173 | 6.841 | 1.000 |
| 0.01 | 0.003 | 0.390 | 270.2 | 0.001 | 0.315 | 231.0 | 0.014 | 0.250 | 681.5 | −0.003 | 0.205 | 48.21 | 0.003 | 0.155 | $1.95 \cdot 10^3$ |
| 0.05 | 0.006 | 0.294 | 473.1 | 0.001 | 0.224 | 460.0 | 0.010 | 0.170 | 1470.2 | −0.002 | 0.132 | 117.5 | 0.001 | 0.094 | $5.24 \cdot 10^3$ |
| 0.10 | 0.006 | 0.266 | 577.7 | −0.000 | 0.195 | 605.6 | 0.008 | 0.144 | 2048.9 | −0.002 | 0.108 | 173.6 | 0.001 | 0.075 | $8.42 \cdot 10^3$ |
| 0.20 | 0.005 | 0.245 | 681.3 | −0.002 | 0.171 | 785.7 | 0.006 | 0.122 | 2844.6 | −0.002 | 0.089 | 258.0 | 0.000 | 0.058 | $1.41 \cdot 10^4$ |
| 0.50 | 0.003 | 0.232 | 765.0 | −0.003 | 0.147 | 1062.3 | 0.005 | 0.099 | 4355.7 | −0.001 | 0.068 | 443.8 | −0.000 | 0.039 | $3.01 \cdot 10^4$ |

Table 106: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.099 | 6.404 | 1.000 | −0.272 | 4.787 | 1.000 | 0.194 | 6.527 | 1.000 | −0.009 | 1.426 | 1.000 | −0.173 | 6.841 | 1.000 |
| 0.01 | 0.006 | 0.389 | 271.3 | 0.007 | 0.315 | 231.4 | 0.021 | 0.251 | 674.7 | 0.007 | 0.206 | 47.94 | 0.019 | 0.157 | $1.87 \cdot 10^3$ |
| 0.05 | 0.013 | 0.295 | 471.5 | 0.012 | 0.225 | 451.0 | 0.025 | 0.172 | 1404.4 | 0.018 | 0.135 | 110.3 | 0.032 | 0.100 | $4.24 \cdot 10^3$ |
| 0.10 | 0.014 | 0.267 | 572.3 | 0.014 | 0.198 | 585.5 | 0.028 | 0.147 | 1894.8 | 0.024 | 0.113 | 152.5 | 0.039 | 0.082 | $5.63 \cdot 10^3$ |
| 0.20 | 0.015 | 0.247 | 669.5 | 0.016 | 0.175 | 746.0 | 0.032 | 0.127 | 2498.6 | 0.032 | 0.095 | 202.3 | 0.048 | 0.067 | $6.84 \cdot 10^3$ |
| 0.50 | 0.014 | 0.235 | 740.7 | 0.021 | 0.152 | 974.0 | 0.040 | 0.105 | 3394.1 | 0.044 | 0.076 | 264.5 | 0.062 | 0.050 | $7.4 \cdot 10^3$ |

Table 107: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.099 | 6.404 | 1.000 | −0.272 | 4.787 | 1.000 | 0.194 | 6.527 | 1.000 | −0.009 | 1.426 | 1.000 | −0.173 | 6.841 | 1.000 |
| 0.01 | 0.009 | 0.388 | 271.7 | 0.012 | 0.316 | 230.5 | 0.027 | 0.253 | 660.6 | 0.017 | 0.208 | 46.49 | 0.034 | 0.164 | $1.68 \cdot 10^3$ |
| 0.05 | 0.019 | 0.296 | 466.1 | 0.022 | 0.230 | 432.2 | 0.039 | 0.178 | 1279.6 | 0.038 | 0.143 | 92.90 | 0.062 | 0.114 | $2.77 \cdot 10^3$ |
| 0.10 | 0.022 | 0.270 | 559.1 | 0.028 | 0.204 | 544.3 | 0.047 | 0.156 | 1610.5 | 0.050 | 0.125 | 112.3 | 0.078 | 0.100 | $2.91 \cdot 10^3$ |
| 0.20 | 0.025 | 0.251 | 643.3 | 0.034 | 0.183 | 663.4 | 0.058 | 0.138 | 1905.8 | 0.065 | 0.111 | 123.2 | 0.097 | 0.088 | $2.74 \cdot 10^3$ |
| 0.50 | 0.025 | 0.243 | 688.7 | 0.045 | 0.164 | 792.4 | 0.075 | 0.121 | 2105.8 | 0.088 | 0.096 | 119.4 | 0.124 | 0.073 | $2.27 \cdot 10^3$ |

Table 108: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.099 | 6.404 | 1.000 | −0.272 | 4.787 | 1.000 | 0.194 | 6.527 | 1.000 | −0.009 | 1.426 | 1.000 | −0.173 | 6.841 | 1.000 |
| 0.01 | 0.018 | 0.390 | 269.6 | 0.027 | 0.322 | 220.8 | 0.048 | 0.265 | 586.0 | 0.046 | 0.228 | 37.69 | 0.080 | 0.200 | $1.01 \cdot 10^3$ |
| 0.05 | 0.036 | 0.306 | 431.1 | 0.053 | 0.253 | 345.0 | 0.083 | 0.212 | 821.1 | 0.095 | 0.190 | 45.02 | 0.151 | 0.183 | 832.9 |
| 0.10 | 0.045 | 0.287 | 487.6 | 0.068 | 0.237 | 379.6 | 0.106 | 0.202 | 817.1 | 0.126 | 0.186 | 40.33 | 0.195 | 0.180 | 664.2 |
| 0.20 | 0.053 | 0.277 | 517.4 | 0.087 | 0.227 | 388.5 | 0.136 | 0.198 | 738.8 | 0.164 | 0.185 | 33.25 | 0.247 | 0.174 | 513.7 |
| 0.50 | 0.055 | 0.287 | 479.1 | 0.115 | 0.228 | 353.1 | 0.182 | 0.200 | 581.5 | 0.225 | 0.183 | 24.12 | 0.322 | 0.153 | 368.4 |

Table 109: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.099 | 6.404 | 1.000 | −0.272 | 4.787 | 1.000 | 0.194 | 6.527 | 1.000 | −0.009 | 1.426 | 1.000 | −0.173 | 6.841 | 1.000 |
| 0.01 | 0.032 | 0.399 | 256.1 | 0.052 | 0.344 | 189.8 | 0.082 | 0.305 | 427.5 | 0.093 | 0.286 | 22.49 | 0.155 | 0.291 | 429.9 |
| 0.05 | 0.064 | 0.339 | 344.2 | 0.102 | 0.316 | 208.5 | 0.156 | 0.302 | 368.9 | 0.191 | 0.303 | 15.85 | 0.312 | 0.329 | 227.9 |
| 0.10 | 0.080 | 0.335 | 344.7 | 0.134 | 0.322 | 188.6 | 0.207 | 0.318 | 296.6 | 0.258 | 0.323 | 11.91 | 0.412 | 0.340 | 163.9 |
| 0.20 | 0.097 | 0.347 | 315.9 | 0.174 | 0.338 | 158.9 | 0.272 | 0.339 | 225.6 | 0.343 | 0.342 | 8.663 | 0.530 | 0.332 | 119.7 |
| 0.50 | 0.104 | 0.401 | 239.3 | 0.234 | 0.377 | 116.8 | 0.375 | 0.372 | 152.9 | 0.481 | 0.353 | 5.713 | 0.695 | 0.280 | 83.44 |

Table 110: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.419 | 8.838 | 1.000 | −0.739 | 16.201 | 1.000 | 0.133 | 7.153 | 1.000 | −0.146 | 2.424 | 1.000 | −0.733 | 17.044 | 1.000 |
| 0.01 | 0.000 | 0.526 | 283.1 | 0.004 | 0.415 | 1528.7 | −0.013 | 0.339 | 445.7 | −0.006 | 0.265 | 84.17 | 0.007 | 0.191 | $7.93 \cdot 10^3$ |
| 0.05 | 0.002 | 0.361 | 600.0 | 0.003 | 0.271 | 3592.7 | −0.009 | 0.207 | 1191.7 | −0.002 | 0.153 | 250.4 | 0.003 | 0.102 | $2.82 \cdot 10^4$ |
| 0.10 | 0.003 | 0.310 | 815.0 | 0.001 | 0.224 | 5253.1 | −0.008 | 0.164 | 1888.8 | −0.001 | 0.117 | 427.4 | 0.002 | 0.073 | $5.46 \cdot 10^4$ |
| 0.20 | 0.003 | 0.269 | 1080.6 | 0.001 | 0.185 | 7701.7 | −0.006 | 0.129 | 3070.4 | −0.001 | 0.088 | 769.1 | 0.001 | 0.050 | $1.15 \cdot 10^5$ |
| 0.50 | 0.004 | 0.234 | 1424.3 | 0.000 | 0.145 | 12543.8 | −0.004 | 0.092 | 6068.2 | −0.001 | 0.057 | 1840.8 | 0.001 | 0.028 | $3.6 \cdot 10^5$ |

Table 111: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.419 | 8.838 | 1.000 | −0.739 | 16.201 | 1.000 | 0.133 | 7.153 | 1.000 | −0.146 | 2.424 | 1.000 | −0.733 | 17.044 | 1.000 |
| 0.01 | 0.007 | 0.526 | 282.8 | 0.014 | 0.415 | 1525.0 | 0.002 | 0.340 | 443.6 | 0.013 | 0.265 | 83.69 | 0.035 | 0.194 | $7.47 \cdot 10^3$ |
| 0.05 | 0.014 | 0.362 | 597.6 | 0.022 | 0.273 | 3517.5 | 0.019 | 0.210 | 1156.2 | 0.033 | 0.156 | 230.9 | 0.053 | 0.106 | $2.06 \cdot 10^4$ |
| 0.10 | 0.019 | 0.311 | 807.9 | 0.027 | 0.227 | 5057.2 | 0.028 | 0.168 | 1765.4 | 0.043 | 0.122 | 353.2 | 0.063 | 0.079 | $2.87 \cdot 10^4$ |
| 0.20 | 0.023 | 0.271 | 1061.6 | 0.032 | 0.188 | 7203.3 | 0.039 | 0.133 | 2652.7 | 0.054 | 0.093 | 508.5 | 0.072 | 0.056 | $3.46 \cdot 10^4$ |
| 0.50 | 0.028 | 0.237 | 1373.9 | 0.041 | 0.150 | 10925.8 | 0.053 | 0.097 | 4167.0 | 0.068 | 0.064 | 684.2 | 0.083 | 0.034 | $3.59 \cdot 10^4$ |

Table 112: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.419 | 8.838 | 1.000 | −0.739 | 16.201 | 1.000 | 0.133 | 7.153 | 1.000 | −0.146 | 2.424 | 1.000 | −0.733 | 17.044 | 1.000 |
| 0.01 | 0.013 | 0.527 | 281.8 | 0.024 | 0.416 | 1511.8 | 0.017 | 0.343 | 435.1 | 0.031 | 0.268 | 80.84 | 0.064 | 0.202 | $6.51 \cdot 10^3$ |
| 0.05 | 0.027 | 0.364 | 588.5 | 0.041 | 0.277 | 3342.5 | 0.047 | 0.217 | 1042.7 | 0.068 | 0.165 | 184.6 | 0.103 | 0.119 | $1.17 \cdot 10^4$ |
| 0.10 | 0.035 | 0.314 | 784.3 | 0.052 | 0.233 | 4602.7 | 0.064 | 0.178 | 1437.6 | 0.088 | 0.134 | 230.9 | 0.123 | 0.093 | $1.21 \cdot 10^4$ |
| 0.20 | 0.043 | 0.276 | 1005.7 | 0.064 | 0.198 | 6091.1 | 0.083 | 0.146 | 1818.6 | 0.108 | 0.108 | 252.4 | 0.143 | 0.071 | $1.14 \cdot 10^4$ |
| 0.50 | 0.051 | 0.246 | 1237.7 | 0.082 | 0.163 | 7890.0 | 0.109 | 0.113 | 2070.8 | 0.136 | 0.080 | 237.7 | 0.166 | 0.046 | $9.77 \cdot 10^3$ |

Table 113: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.419 | 8.838 | 1.000 | −0.739 | 16.201 | 1.000 | 0.133 | 7.153 | 1.000 | −0.146 | 2.424 | 1.000 | −0.733 | 17.044 | 1.000 |
| 0.01 | 0.033 | 0.533 | 274.3 | 0.054 | 0.427 | 1422.4 | 0.061 | 0.361 | 381.5 | 0.086 | 0.292 | 63.88 | 0.148 | 0.245 | $3.55 \cdot 10^3$ |
| 0.05 | 0.063 | 0.380 | 528.4 | 0.099 | 0.307 | 2524.3 | 0.129 | 0.259 | 609.7 | 0.171 | 0.217 | 77.28 | 0.252 | 0.186 | $2.96 \cdot 10^3$ |
| 0.10 | 0.081 | 0.338 | 649.4 | 0.125 | 0.275 | 2885.2 | 0.169 | 0.233 | 618.3 | 0.219 | 0.197 | 68.10 | 0.306 | 0.162 | $2.43 \cdot 10^3$ |
| 0.20 | 0.100 | 0.310 | 738.3 | 0.157 | 0.252 | 2989.6 | 0.213 | 0.213 | 563.5 | 0.272 | 0.178 | 55.92 | 0.360 | 0.133 | $1.98 \cdot 10^3$ |
| 0.50 | 0.118 | 0.302 | 742.4 | 0.204 | 0.236 | 2714.3 | 0.276 | 0.191 | 453.1 | 0.343 | 0.149 | 42.18 | 0.420 | 0.090 | $1.57 \cdot 10^3$ |

Table 114: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.419 | 8.838 | 1.000 | −0.739 | 16.201 | 1.000 | 0.133 | 7.153 | 1.000 | −0.146 | 2.424 | 1.000 | −0.733 | 17.044 | 1.000 |
| 0.01 | 0.065 | 0.556 | 250.1 | 0.101 | 0.462 | 1177.2 | 0.132 | 0.420 | 264.7 | 0.173 | 0.364 | 36.41 | 0.287 | 0.358 | $1.38 \cdot 10^3$ |
| 0.05 | 0.122 | 0.433 | 387.3 | 0.192 | 0.393 | 1376.6 | 0.264 | 0.372 | 245.6 | 0.342 | 0.344 | 25.08 | 0.511 | 0.328 | 788.8 |
| 0.10 | 0.155 | 0.411 | 405.2 | 0.247 | 0.385 | 1258.4 | 0.342 | 0.368 | 202.6 | 0.445 | 0.337 | 18.93 | 0.630 | 0.293 | 602.5 |
| 0.20 | 0.190 | 0.409 | 384.7 | 0.312 | 0.386 | 1066.1 | 0.435 | 0.364 | 159.4 | 0.560 | 0.320 | 14.17 | 0.746 | 0.239 | 474.5 |
| 0.50 | 0.224 | 0.448 | 312.4 | 0.409 | 0.402 | 800.7 | 0.569 | 0.350 | 114.7 | 0.713 | 0.273 | 10.13 | 0.867 | 0.153 | 375.6 |

Table 115: $\text{Max}\underline{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = 1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.411 | 24.225 | 1.000 | 0.010 | 25.109 | 1.000 | 0.209 | 6.925 | 1.000 | −0.153 | 13.123 | 1.000 | −0.420 | 18.989 | 1.000 |
| 0.01 | −0.017 | 0.587 | 1699.6 | 0.014 | 0.482 | 2711.6 | 0.002 | 0.384 | 325.9 | 0.004 | 0.300 | 1917.9 | 0.002 | 0.214 | $7.85 \cdot 10^3$ |
| 0.05 | −0.014 | 0.386 | 3939.8 | 0.006 | 0.297 | 7157.3 | −0.002 | 0.223 | 968.7 | 0.002 | 0.163 | 6491.1 | −0.000 | 0.103 | $3.41 \cdot 10^4$ |
| 0.10 | −0.012 | 0.322 | 5652.7 | 0.003 | 0.237 | 11251.1 | −0.002 | 0.170 | 1666.4 | 0.001 | 0.119 | 12123.8 | −0.001 | 0.070 | $7.42 \cdot 10^4$ |
| 0.20 | −0.010 | 0.270 | 8026.0 | 0.002 | 0.187 | 18068.0 | −0.002 | 0.126 | 3037.6 | 0.001 | 0.084 | 24478.8 | −0.001 | 0.045 | $1.79 \cdot 10^5$ |
| 0.50 | −0.008 | 0.221 | 12026.0 | 0.001 | 0.135 | 34457.4 | −0.001 | 0.081 | 7385.5 | 0.001 | 0.049 | 70913.3 | −0.001 | 0.023 | $6.75 \cdot 10^5$ |

Table 116: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.411 | 24.225 | 1.000 | 0.010 | 25.109 | 1.000 | 0.209 | 6.925 | 1.000 | −0.153 | 13.123 | 1.000 | −0.420 | 18.989 | 1.000 |
| 0.01 | −0.008 | 0.588 | 1697.3 | 0.028 | 0.483 | 2688.8 | 0.023 | 0.386 | 321.4 | 0.030 | 0.301 | 1880.5 | 0.040 | 0.217 | $7.4 \cdot 10^3$ |
| 0.05 | 0.003 | 0.387 | 3928.7 | 0.033 | 0.299 | 6961.1 | 0.036 | 0.226 | 919.5 | 0.049 | 0.166 | 5752.9 | 0.062 | 0.108 | $2.33 \cdot 10^4$ |
| 0.10 | 0.010 | 0.323 | 5609.7 | 0.038 | 0.240 | 10704.9 | 0.046 | 0.173 | 1494.0 | 0.059 | 0.123 | 9290.9 | 0.072 | 0.075 | $3.34 \cdot 10^4$ |
| 0.20 | 0.017 | 0.272 | 7874.7 | 0.045 | 0.190 | 16464.7 | 0.056 | 0.130 | 2398.8 | 0.069 | 0.088 | 13827.3 | 0.081 | 0.050 | $4.01 \cdot 10^4$ |
| 0.50 | 0.026 | 0.225 | 11430.9 | 0.056 | 0.140 | 27801.9 | 0.069 | 0.085 | 3972.6 | 0.081 | 0.053 | 18408.2 | 0.090 | 0.027 | $4.12 \cdot 10^4$ |

Table 117: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.1$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.411 | 24.225 | 1.000 | 0.010 | 25.109 | 1.000 | 0.209 | 6.925 | 1.000 | −0.153 | 13.123 | 1.000 | −0.420 | 18.989 | 1.000 |
| 0.01 | 0.000 | 0.589 | 1690.2 | 0.042 | 0.486 | 2647.9 | 0.043 | 0.390 | 312.1 | 0.056 | 0.306 | 1785.5 | 0.079 | 0.224 | $6.38 \cdot 10^3$ |
| 0.05 | 0.019 | 0.389 | 3865.2 | 0.060 | 0.305 | 6532.2 | 0.074 | 0.233 | 802.5 | 0.096 | 0.175 | 4338.9 | 0.125 | 0.119 | $1.21 \cdot 10^4$ |
| 0.10 | 0.031 | 0.327 | 5424.4 | 0.073 | 0.247 | 9492.3 | 0.093 | 0.183 | 1141.0 | 0.116 | 0.133 | 5516.6 | 0.145 | 0.087 | $1.27 \cdot 10^4$ |
| 0.20 | 0.044 | 0.279 | 7362.5 | 0.089 | 0.200 | 13173.0 | 0.113 | 0.141 | 1463.6 | 0.137 | 0.099 | 6042.6 | 0.162 | 0.060 | $1.2 \cdot 10^4$ |
| 0.50 | 0.059 | 0.237 | 9859.0 | 0.111 | 0.153 | 17730.8 | 0.139 | 0.098 | 1650.3 | 0.161 | 0.063 | 5745.3 | 0.180 | 0.034 | $1.08 \cdot 10^4$ |

Table 118: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.2$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.411 | 24.225 | 1.000 | 0.010 | 25.109 | 1.000 | 0.209 | 6.925 | 1.000 | −0.153 | 13.123 | 1.000 | −0.420 | 18.989 | 1.000 |
| 0.01 | 0.026 | 0.597 | 1642.7 | 0.085 | 0.501 | 2436.6 | 0.104 | 0.413 | 264.8 | 0.133 | 0.334 | 1334.5 | 0.193 | 0.268 | $3.31 \cdot 10^3$ |
| 0.05 | 0.069 | 0.408 | 3428.3 | 0.143 | 0.339 | 4659.8 | 0.187 | 0.277 | 428.7 | 0.234 | 0.226 | 1623.1 | 0.311 | 0.178 | $2.81 \cdot 10^3$ |
| 0.10 | 0.095 | 0.355 | 4347.7 | 0.178 | 0.292 | 5394.0 | 0.234 | 0.237 | 433.1 | 0.287 | 0.190 | 1453.4 | 0.362 | 0.142 | $2.38 \cdot 10^3$ |
| 0.20 | 0.123 | 0.319 | 5024.9 | 0.218 | 0.256 | 5585.9 | 0.285 | 0.202 | 394.1 | 0.340 | 0.155 | 1230.1 | 0.407 | 0.106 | $2.04 \cdot 10^3$ |
| 0.50 | 0.156 | 0.302 | 5072.8 | 0.273 | 0.222 | 5078.2 | 0.350 | 0.161 | 323.5 | 0.403 | 0.111 | 985.2 | 0.452 | 0.063 | $1.73 \cdot 10^3$ |

Table 119: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 0.5$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.411 | 24.225 | 1.000 | 0.010 | 25.109 | 1.000 | 0.209 | 6.925 | 1.000 | −0.153 | 13.123 | 1.000 | −0.420 | 18.989 | 1.000 |
| 0.01 | 0.068 | 0.623 | 1494.5 | 0.156 | 0.548 | 1943.6 | 0.205 | 0.480 | 176.0 | 0.259 | 0.417 | 714.0 | 0.382 | 0.383 | $1.23 \cdot 10^3$ |
| 0.05 | 0.149 | 0.468 | 2433.4 | 0.278 | 0.436 | 2361.5 | 0.373 | 0.394 | 163.0 | 0.466 | 0.353 | 503.4 | 0.628 | 0.306 | 738.7 |
| 0.10 | 0.196 | 0.438 | 2546.2 | 0.349 | 0.412 | 2162.4 | 0.469 | 0.368 | 134.9 | 0.578 | 0.318 | 395.7 | 0.737 | 0.251 | 595.7 |
| 0.20 | 0.249 | 0.430 | 2380.1 | 0.431 | 0.395 | 1843.2 | 0.575 | 0.339 | 107.8 | 0.692 | 0.271 | 312.0 | 0.830 | 0.186 | 498.4 |
| 0.50 | 0.309 | 0.461 | 1907.8 | 0.546 | 0.381 | 1423.4 | 0.711 | 0.290 | 81.46 | 0.821 | 0.198 | 241.2 | 0.917 | 0.105 | 423.3 |

Table 120: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = 1$

### 4.3 Tabulated results for power subdivergence estimators escorted by MLE

In accordance with the fact that the only reasonable results we obtained were for $\mu = 0$ which is the true parameter of the estimated data, the best results were received for the value of the escort parameter $\mu = \bar{\boldsymbol{X}}_n$ as was already indicated by theory in [3].

For contamination by $N(0,9)$, $N(0,100)$ and $Lo(0,1)$ (cf. tables 121 - 138) we received perfect match with MLE for all values of $\varepsilon$. Nevertheless, an **outstanding** behavior was noticed in case of contamination by Cauchy distribution, where the power subdivergence estimator shows a **significant** resistance to distant outliers ( cf. Figure 9 and tables 139 - 144). In the situation when the standard deviation of the maximum likelihood estimator with great volatility copies the occurrence of extreme outliers, the standard deviation of MLE escorted subdivergence estimator retains low values and steady convergence to 0. This, clearly, results also in huge empirical relative efficiency (cf. e.g. Table 144)

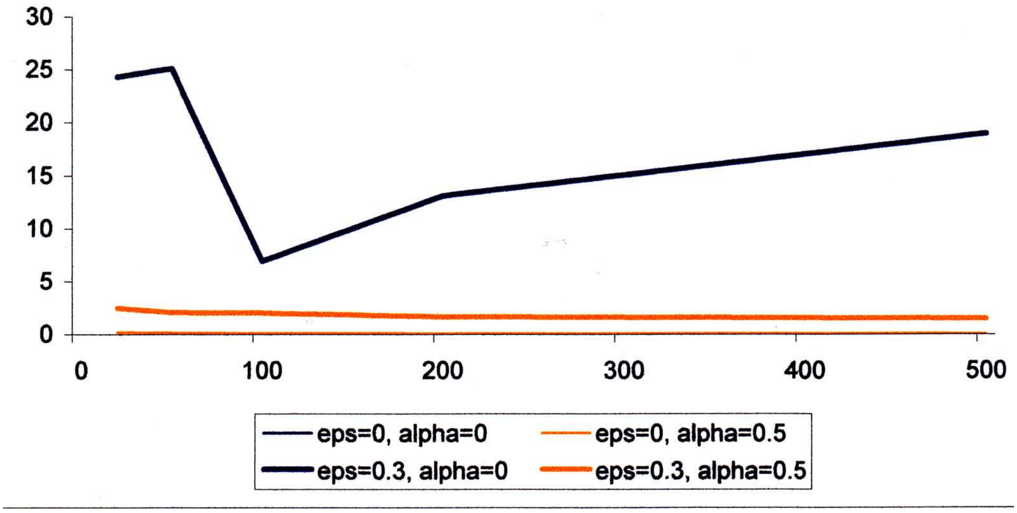


Figure 9: Dependency of standard deviation of the max$\underline{\text{D}}_\alpha$-estimators with escort parameter $\mu = \bar{\boldsymbol{X}}_n$ on sample size $n$ for data distributed by $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.20 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.50 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |

Table 121: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.01 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.05 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.10 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.20 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |
| 0.50 | 0.011 | 0.226 | 1.000 | −0.004 | 0.145 | 1.000 | 0.002 | 0.099 | 1.000 | −0.002 | 0.077 | 1.000 | 0.002 | 0.045 | 1.000 |

Table 122: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.01 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.05 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.10 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.20 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |
| 0.50 | 0.014 | 0.257 | 1.000 | −0.001 | 0.168 | 1.000 | 0.003 | 0.113 | 1.000 | −0.004 | 0.088 | 1.000 | 0.002 | 0.051 | 1.000 |

Table 123: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.01 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.05 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.10 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.20 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |
| 0.50 | 0.015 | 0.284 | 1.000 | −0.003 | 0.190 | 1.000 | 0.001 | 0.131 | 1.000 | −0.005 | 0.099 | 1.000 | 0.001 | 0.059 | 1.000 |

Table 124: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.01 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.05 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.10 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.20 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |
| 0.50 | 0.017 | 0.350 | 1.000 | −0.001 | 0.228 | 1.000 | 0.002 | 0.157 | 1.000 | −0.005 | 0.120 | 1.000 | 0.001 | 0.072 | 1.000 |

Table 125: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.01 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.05 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.10 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.20 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |
| 0.50 | 0.014 | 0.401 | 1.000 | 0.001 | 0.259 | 1.000 | 0.002 | 0.176 | 1.000 | −0.005 | 0.137 | 1.000 | 0.001 | 0.083 | 1.000 |

Table 126: Max$\underline{D}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.20 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.50 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |

Table 127: MaxD̰$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.01 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.05 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.10 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.20 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |
| 0.50 | 0.010 | 0.293 | 1.000 | −0.002 | 0.192 | 1.000 | 0.003 | 0.135 | 1.000 | −0.001 | 0.104 | 1.000 | 0.002 | 0.061 | 1.000 |

Table 128: MaxD̰$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.01 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.05 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.10 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.20 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |
| 0.50 | 0.024 | 0.523 | 1.000 | 0.011 | 0.346 | 1.000 | 0.007 | 0.236 | 1.000 | −0.009 | 0.181 | 1.000 | 0.004 | 0.103 | 1.000 |

Table 129: MaxD̰$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.01 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.05 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.10 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.20 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |
| 0.50 | 0.032 | 0.676 | 1.000 | 0.003 | 0.479 | 1.000 | 0.000 | 0.332 | 1.000 | −0.013 | 0.239 | 1.000 | 0.002 | 0.146 | 1.000 |

Table 130: MaxD̰$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.01 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.05 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.10 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.20 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |
| 0.50 | 0.038 | 1.001 | 1.000 | 0.011 | 0.656 | 1.000 | 0.005 | 0.455 | 1.000 | −0.011 | 0.336 | 1.000 | 0.001 | 0.205 | 1.000 |

Table 131: MaxD$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.01 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.05 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.10 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.20 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |
| 0.50 | 0.027 | 1.219 | 1.000 | 0.018 | 0.780 | 1.000 | 0.002 | 0.534 | 1.000 | −0.014 | 0.406 | 1.000 | −0.002 | 0.252 | 1.000 |

Table 132: MaxD$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.20 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.50 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |

Table 133: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.01 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.05 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.10 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.20 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |
| 0.50 | −0.000 | 0.217 | 1.000 | −0.002 | 0.145 | 1.000 | 0.005 | 0.101 | 1.000 | −0.001 | 0.071 | 1.000 | 0.000 | 0.045 | 1.000 |

Table 134: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.01 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.05 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.10 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.20 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |
| 0.50 | −0.010 | 0.235 | 1.000 | −0.002 | 0.148 | 1.000 | 0.001 | 0.104 | 1.000 | 0.002 | 0.074 | 1.000 | −0.001 | 0.046 | 1.000 |

Table 135: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.01 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.05 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.10 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.20 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |
| 0.50 | 0.005 | 0.240 | 1.000 | −0.003 | 0.161 | 1.000 | 0.004 | 0.112 | 1.000 | −0.000 | 0.079 | 1.000 | −0.001 | 0.050 | 1.000 |

Table 136: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.01 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.05 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.10 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.20 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |
| 0.50 | 0.004 | 0.269 | 1.000 | −0.004 | 0.178 | 1.000 | −0.004 | 0.124 | 1.000 | −0.002 | 0.084 | 1.000 | 0.002 | 0.054 | 1.000 |

Table 137: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.01 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.05 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.10 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.20 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |
| 0.50 | −0.008 | 0.277 | 1.000 | −0.000 | 0.192 | 1.000 | −0.001 | 0.134 | 1.000 | −0.001 | 0.092 | 1.000 | 0.000 | 0.057 | 1.000 |

Table 138: MaxD̲$_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.01 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.05 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.10 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.20 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |
| 0.50 | 0.011 | 0.218 | 1.000 | −0.004 | 0.140 | 1.000 | 0.001 | 0.095 | 1.000 | −0.003 | 0.073 | 1.000 | 0.001 | 0.044 | 1.000 |

Table 139: $\mathrm{Max}\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon C(0,1)$, $\varepsilon = 0$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.01 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.05 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.10 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.20 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |
| 0.50 | 0.004 | 0.368 | 1.000 | 0.010 | 0.364 | 1.000 | 0.027 | 0.582 | 1.000 | 0.006 | 0.411 | 1.000 | 0.005 | 0.162 | 1.000 |

Table 140: $\mathrm{Max}\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon C(0,1)$, $\varepsilon = 0.01$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.689 | 16.85 | 1.000 |
| 0.01 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.001 | 1.349 | 156.3 |
| 0.05 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | 0.024 | 1.368 | 1.000 | −0.037 | 0.947 | 316.6 |
| 0.10 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.096 | 1.864 | 1.000 | −0.009 | 1.040 | 1.729 | −0.018 | 0.590 | 817.1 |
| 0.20 | −0.078 | 2.622 | 1.000 | 0.009 | 0.735 | 1.000 | −0.040 | 1.168 | 2.551 | −0.009 | 1.040 | 1.729 | −0.018 | 0.590 | 817.1 |
| 0.50 | −0.084 | 1.241 | 4.450 | 0.009 | 0.735 | 1.000 | −0.020 | 0.895 | 4.349 | −0.020 | 0.806 | 2.878 | −0.009 | 0.581 | 840.8 |

Table 141: $\mathrm{Max}\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon C(0,1)$, $\varepsilon = 0.05$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.099 | 6.404 | 1.000 | −0.272 | 4.787 | 1.000 | 0.194 | 6.527 | 1.000 | −0.009 | 1.426 | 1.000 | −0.173 | 6.841 | 1.000 |
| 0.01 | −0.099 | 6.404 | 1.000 | −0.272 | 4.787 | 1.000 | 0.001 | 2.682 | 5.929 | −0.009 | 1.426 | 1.000 | 0.031 | 1.856 | 13.59 |
| 0.05 | 0.069 | 3.339 | 3.678 | −0.143 | 2.275 | 4.426 | −0.065 | 1.857 | 12.34 | −0.009 | 1.426 | 1.000 | −0.003 | 1.620 | 17.84 |
| 0.10 | 0.069 | 3.339 | 3.678 | −0.143 | 2.275 | 4.426 | −0.065 | 1.857 | 12.34 | −0.042 | 1.114 | 1.637 | −0.008 | 1.316 | 27.05 |
| 0.20 | 0.001 | 2.666 | 5.771 | −0.097 | 1.622 | 8.711 | −0.034 | 1.474 | 19.62 | −0.042 | 1.114 | 1.637 | −0.052 | 1.103 | 38.41 |
| 0.50 | −0.003 | 1.386 | 21.36 | −0.066 | 1.148 | 17.38 | 0.005 | 1.006 | 42.09 | −0.053 | 0.901 | 2.496 | −0.039 | 1.007 | 46.16 |

Table 142: $\mathrm{Max}\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon C(0,1)$, $\varepsilon = 0.1$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | 0.419 | 8.838 | 1.000 | −0.739 | 16.20 | 1.000 | 0.133 | 7.153 | 1.000 | −0.146 | 2.424 | 1.000 | −0.733 | 17.04 | 1.000 |
| 0.01 | 0.419 | 8.838 | 1.000 | −0.254 | 4.769 | 11.53 | −0.061 | 3.968 | 3.250 | −0.146 | 2.424 | 1.000 | −0.045 | 2.914 | 34.27 |
| 0.05 | 0.093 | 5.301 | 2.786 | −0.128 | 2.378 | 46.38 | −0.127 | 3.460 | 4.270 | −0.107 | 1.977 | 1.504 | −0.102 | 2.148 | 62.92 |
| 0.10 | 0.093 | 5.301 | 2.786 | −0.128 | 2.378 | 46.38 | −0.103 | 2.509 | 8.118 | −0.083 | 1.767 | 1.886 | −0.075 | 1.699 | 100.6 |
| 0.20 | −0.046 | 3.278 | 7.283 | −0.128 | 2.378 | 46.38 | −0.043 | 1.973 | 13.15 | −0.046 | 1.436 | 2.859 | −0.075 | 1.482 | 132.2 |
| 0.50 | −0.051 | 2.293 | 14.88 | −0.053 | 1.746 | 86.22 | −0.033 | 1.621 | 19.47 | −0.060 | 1.244 | 3.802 | −0.068 | 1.439 | 140.2 |

Table 143: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.2$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ | $m(\tilde{\mu})$ | $s(\tilde{\mu})$ | $eref(\tilde{\mu})$ |
| 0.00 | −0.411 | 24.23 | 1.000 | 0.010 | 25.11 | 1.000 | 0.209 | 6.925 | 1.000 | −0.153 | 13.12 | 1.000 | −0.420 | 18.99 | 1.000 |
| 0.01 | 0.298 | 8.752 | 7.655 | −0.094 | 7.137 | 12.38 | 0.016 | 3.566 | 3.775 | −0.019 | 4.657 | 7.942 | 0.003 | 3.239 | 34.39 |
| 0.05 | −0.028 | 5.159 | 22.06 | −0.063 | 5.061 | 24.61 | 0.016 | 3.566 | 3.775 | −0.125 | 2.345 | 31.24 | −0.051 | 2.177 | 76.05 |
| 0.10 | −0.028 | 5.159 | 22.06 | −0.065 | 4.266 | 34.64 | −0.083 | 2.972 | 5.429 | −0.159 | 2.157 | 36.83 | −0.045 | 1.931 | 96.69 |
| 0.20 | −0.101 | 3.601 | 45.24 | −0.072 | 3.228 | 60.47 | 0.032 | 2.096 | 10.93 | −0.137 | 1.984 | 43.56 | −0.083 | 1.631 | 135.2 |
| 0.50 | −0.073 | 2.489 | 94.66 | −0.070 | 2.060 | 148.4 | 0.008 | 2.018 | 11.78 | −0.150 | 1.648 | 62.91 | −0.102 | 1.485 | 162.9 |

Table 144: Max$\underline{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.3$, $K = 1000$, $\mu = \bar{\boldsymbol{X}}_n$

## 4.4 Tabulated results for power superdivergence estimators

This section is to summarize the results received by simulating power superdivergence estimators.

As well as in case of max$\underline{\mathrm{D}}_\alpha$-estimators escorted by $\mu = \bar{\boldsymbol{X}}_n$ we received a perfect match with maximum likelihood estimatior for all mixtures (cf. tables 145 - 162) except for the mixture $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$ (cf. tables 163 - 168). In this case again, with higher contamination the min$\bar{\mathrm{D}}_\alpha$-estimators show favourable robusness and rising efficiency. Yet, these robustness tendencies are not as strong as with power subdivergence estimators escorted by MLE.

Another demotivating feature of superdivergence estimators computation is extremely high computing time caused by double optimization. This price is too high to pay for the above mentioned robustness, and it strongly discourages the users from further utilization.

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.01 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.05 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.10 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.20 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.50 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |

Table 145: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.007 | 0.218 | 1.000 | 0.032 | 0.158 | 1.000 | 0.004 | 0.103 | 1.000 | 0.000 | 0.068 | 1.000 | −0.003 | 0.043 | 1.000 |
| 0.01 | −0.007 | 0.218 | 1.000 | 0.032 | 0.158 | 1.000 | 0.004 | 0.103 | 1.000 | 0.000 | 0.068 | 1.000 | −0.003 | 0.043 | 1.000 |
| 0.05 | −0.007 | 0.218 | 1.000 | 0.032 | 0.158 | 1.000 | 0.004 | 0.103 | 1.000 | 0.000 | 0.068 | 1.000 | −0.003 | 0.043 | 1.000 |
| 0.10 | −0.007 | 0.218 | 1.000 | 0.032 | 0.158 | 1.000 | 0.004 | 0.103 | 1.000 | 0.000 | 0.068 | 1.000 | −0.003 | 0.043 | 1.000 |
| 0.20 | −0.007 | 0.218 | 1.000 | 0.032 | 0.158 | 1.000 | 0.004 | 0.103 | 1.000 | 0.000 | 0.068 | 1.000 | −0.003 | 0.043 | 1.000 |
| 0.50 | −0.007 | 0.218 | 1.000 | 0.032 | 0.158 | 1.000 | 0.004 | 0.103 | 1.000 | 0.000 | 0.068 | 1.000 | −0.003 | 0.043 | 1.000 |

Table 146: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.01$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.007 | 0.253 | 1.000 | 0.031 | 0.178 | 1.000 | 0.008 | 0.114 | 1.000 | 0.006 | 0.081 | 1.000 | −0.002 | 0.049 | 1.000 |
| 0.01 | −0.007 | 0.253 | 1.000 | 0.031 | 0.178 | 1.000 | 0.008 | 0.114 | 1.000 | 0.006 | 0.081 | 1.000 | −0.002 | 0.049 | 1.000 |
| 0.05 | −0.007 | 0.253 | 1.000 | 0.031 | 0.178 | 1.000 | 0.008 | 0.114 | 1.000 | 0.006 | 0.081 | 1.000 | −0.002 | 0.049 | 1.000 |
| 0.10 | −0.007 | 0.253 | 1.000 | 0.031 | 0.178 | 1.000 | 0.008 | 0.114 | 1.000 | 0.006 | 0.081 | 1.000 | −0.002 | 0.049 | 1.000 |
| 0.20 | −0.007 | 0.253 | 1.000 | 0.031 | 0.178 | 1.000 | 0.008 | 0.114 | 1.000 | 0.006 | 0.081 | 1.000 | −0.002 | 0.049 | 1.000 |
| 0.50 | −0.007 | 0.253 | 1.000 | 0.031 | 0.178 | 1.000 | 0.008 | 0.114 | 1.000 | 0.006 | 0.081 | 1.000 | −0.002 | 0.049 | 1.000 |

Table 147: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.05$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.002 | 0.269 | 1.000 | 0.050 | 0.202 | 1.000 | 0.005 | 0.124 | 1.000 | −0.002 | 0.091 | 1.000 | −0.002 | 0.058 | 1.000 |
| 0.01 | −0.002 | 0.269 | 1.000 | 0.050 | 0.202 | 1.000 | 0.005 | 0.124 | 1.000 | −0.002 | 0.091 | 1.000 | −0.002 | 0.058 | 1.000 |
| 0.05 | −0.002 | 0.269 | 1.000 | 0.050 | 0.202 | 1.000 | 0.005 | 0.124 | 1.000 | −0.002 | 0.091 | 1.000 | −0.002 | 0.058 | 1.000 |
| 0.10 | −0.002 | 0.269 | 1.000 | 0.050 | 0.202 | 1.000 | 0.005 | 0.124 | 1.000 | −0.002 | 0.091 | 1.000 | −0.002 | 0.058 | 1.000 |
| 0.20 | −0.002 | 0.269 | 1.000 | 0.050 | 0.202 | 1.000 | 0.005 | 0.124 | 1.000 | −0.002 | 0.091 | 1.000 | −0.002 | 0.058 | 1.000 |
| 0.50 | −0.002 | 0.269 | 1.000 | 0.050 | 0.202 | 1.000 | 0.005 | 0.124 | 1.000 | −0.002 | 0.091 | 1.000 | −0.002 | 0.058 | 1.000 |

Table 148: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.1$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.015 | 0.320 | 1.000 | 0.050 | 0.261 | 1.000 | 0.004 | 0.159 | 1.000 | −0.007 | 0.117 | 1.000 | 0.003 | 0.073 | 1.000 |
| 0.01 | 0.015 | 0.320 | 1.000 | 0.050 | 0.261 | 1.000 | 0.004 | 0.159 | 1.000 | −0.007 | 0.117 | 1.000 | 0.003 | 0.073 | 1.000 |
| 0.05 | 0.015 | 0.320 | 1.000 | 0.050 | 0.261 | 1.000 | 0.004 | 0.159 | 1.000 | −0.007 | 0.117 | 1.000 | 0.003 | 0.073 | 1.000 |
| 0.10 | 0.015 | 0.320 | 1.000 | 0.050 | 0.261 | 1.000 | 0.004 | 0.159 | 1.000 | −0.007 | 0.117 | 1.000 | 0.003 | 0.073 | 1.000 |
| 0.20 | 0.015 | 0.320 | 1.000 | 0.050 | 0.261 | 1.000 | 0.004 | 0.159 | 1.000 | −0.007 | 0.117 | 1.000 | 0.003 | 0.073 | 1.000 |
| 0.50 | 0.015 | 0.320 | 1.000 | 0.050 | 0.261 | 1.000 | 0.004 | 0.159 | 1.000 | −0.007 | 0.117 | 1.000 | 0.003 | 0.073 | 1.000 |

Table 149: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.2$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.018 | 0.361 | 1.000 | 0.048 | 0.311 | 1.000 | −0.002 | 0.173 | 1.000 | −0.003 | 0.134 | 1.000 | 0.003 | 0.081 | 1.000 |
| 0.01 | 0.018 | 0.361 | 1.000 | 0.048 | 0.311 | 1.000 | −0.002 | 0.173 | 1.000 | −0.003 | 0.134 | 1.000 | 0.003 | 0.081 | 1.000 |
| 0.05 | 0.018 | 0.361 | 1.000 | 0.048 | 0.311 | 1.000 | −0.002 | 0.173 | 1.000 | −0.003 | 0.134 | 1.000 | 0.003 | 0.081 | 1.000 |
| 0.10 | 0.018 | 0.361 | 1.000 | 0.048 | 0.311 | 1.000 | −0.002 | 0.173 | 1.000 | −0.003 | 0.134 | 1.000 | 0.003 | 0.081 | 1.000 |
| 0.20 | 0.018 | 0.361 | 1.000 | 0.048 | 0.311 | 1.000 | −0.002 | 0.173 | 1.000 | −0.003 | 0.134 | 1.000 | 0.003 | 0.081 | 1.000 |
| 0.50 | 0.018 | 0.361 | 1.000 | 0.048 | 0.311 | 1.000 | −0.002 | 0.173 | 1.000 | −0.003 | 0.134 | 1.000 | 0.003 | 0.081 | 1.000 |

Table 150: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,9)$, $\varepsilon = 0.3$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.01 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.05 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.10 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.20 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.50 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |

Table 151: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.015 | 0.261 | 1.000 | 0.037 | 0.204 | 1.000 | 0.001 | 0.144 | 1.000 | 0.009 | 0.087 | 1.000 | −0.004 | 0.060 | 1.000 |
| 0.01 | −0.015 | 0.261 | 1.000 | 0.037 | 0.204 | 1.000 | 0.001 | 0.144 | 1.000 | 0.009 | 0.087 | 1.000 | −0.004 | 0.060 | 1.000 |
| 0.05 | −0.015 | 0.261 | 1.000 | 0.037 | 0.204 | 1.000 | 0.001 | 0.144 | 1.000 | 0.009 | 0.087 | 1.000 | −0.004 | 0.060 | 1.000 |
| 0.10 | −0.015 | 0.261 | 1.000 | 0.037 | 0.204 | 1.000 | 0.001 | 0.144 | 1.000 | 0.009 | 0.087 | 1.000 | −0.004 | 0.060 | 1.000 |
| 0.20 | −0.015 | 0.261 | 1.000 | 0.037 | 0.204 | 1.000 | 0.001 | 0.144 | 1.000 | 0.009 | 0.087 | 1.000 | −0.004 | 0.060 | 1.000 |
| 0.50 | −0.015 | 0.261 | 1.000 | 0.037 | 0.204 | 1.000 | 0.001 | 0.144 | 1.000 | 0.009 | 0.087 | 1.000 | −0.004 | 0.060 | 1.000 |

Table 152: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.01$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.014 | 0.516 | 1.000 | 0.036 | 0.347 | 1.000 | 0.020 | 0.230 | 1.000 | 0.034 | 0.182 | 1.000 | 0.003 | 0.100 | 1.000 |
| 0.01 | −0.014 | 0.516 | 1.000 | 0.036 | 0.347 | 1.000 | 0.020 | 0.230 | 1.000 | 0.034 | 0.182 | 1.000 | 0.003 | 0.100 | 1.000 |
| 0.05 | −0.014 | 0.516 | 1.000 | 0.036 | 0.347 | 1.000 | 0.020 | 0.230 | 1.000 | 0.034 | 0.182 | 1.000 | 0.003 | 0.100 | 1.000 |
| 0.10 | −0.014 | 0.516 | 1.000 | 0.036 | 0.347 | 1.000 | 0.020 | 0.230 | 1.000 | 0.034 | 0.182 | 1.000 | 0.003 | 0.100 | 1.000 |
| 0.20 | −0.014 | 0.516 | 1.000 | 0.036 | 0.347 | 1.000 | 0.020 | 0.230 | 1.000 | 0.034 | 0.182 | 1.000 | 0.003 | 0.100 | 1.000 |
| 0.50 | −0.014 | 0.516 | 1.000 | 0.036 | 0.347 | 1.000 | 0.020 | 0.230 | 1.000 | 0.034 | 0.182 | 1.000 | 0.003 | 0.100 | 0.999 |

Table 153: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.05$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.006 | 0.591 | 1.000 | 0.119 | 0.481 | 1.000 | 0.008 | 0.290 | 1.000 | 0.001 | 0.241 | 1.000 | 0.002 | 0.147 | 1.000 |
| 0.01 | 0.006 | 0.592 | 1.000 | 0.119 | 0.481 | 1.000 | 0.008 | 0.290 | 1.000 | 0.001 | 0.241 | 1.000 | 0.002 | 0.147 | 1.000 |
| 0.05 | 0.006 | 0.592 | 1.000 | 0.119 | 0.481 | 1.000 | 0.008 | 0.290 | 1.000 | 0.001 | 0.241 | 1.000 | 0.002 | 0.147 | 1.000 |
| 0.10 | 0.006 | 0.592 | 1.000 | 0.119 | 0.481 | 1.000 | 0.008 | 0.290 | 1.000 | 0.001 | 0.241 | 1.000 | 0.002 | 0.147 | 1.000 |
| 0.20 | 0.006 | 0.592 | 1.000 | 0.119 | 0.481 | 1.000 | 0.008 | 0.290 | 1.000 | 0.001 | 0.241 | 1.000 | 0.002 | 0.147 | 0.999 |
| 0.50 | 0.006 | 0.592 | 1.000 | 0.119 | 0.481 | 1.000 | 0.008 | 0.290 | 1.000 | 0.000 | 0.241 | 1.000 | 0.002 | 0.147 | 1.000 |

Table 154: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.1$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.085 | 0.850 | 1.000 | 0.120 | 0.752 | 1.000 | 0.002 | 0.458 | 1.000 | −0.022 | 0.356 | 1.000 | 0.024 | 0.218 | 1.000 |
| 0.01 | 0.085 | 0.850 | 1.000 | 0.119 | 0.752 | 1.000 | 0.002 | 0.458 | 1.000 | −0.022 | 0.356 | 1.000 | 0.024 | 0.218 | 1.000 |
| 0.05 | 0.085 | 0.850 | 1.000 | 0.120 | 0.752 | 1.000 | 0.002 | 0.458 | 1.000 | −0.022 | 0.356 | 1.000 | 0.024 | 0.219 | 1.000 |
| 0.10 | 0.085 | 0.850 | 1.000 | 0.119 | 0.752 | 1.000 | 0.002 | 0.458 | 1.000 | −0.022 | 0.356 | 1.000 | 0.024 | 0.218 | 1.000 |
| 0.20 | 0.085 | 0.850 | 1.000 | 0.120 | 0.752 | 1.000 | 0.002 | 0.458 | 1.000 | −0.022 | 0.356 | 1.000 | 0.024 | 0.218 | 1.000 |
| 0.50 | 0.085 | 0.850 | 1.000 | 0.119 | 0.753 | 1.000 | 0.002 | 0.458 | 1.000 | −0.022 | 0.356 | 1.000 | 0.024 | 0.219 | 1.000 |

Table 155: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.2$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.098 | 1.042 | 1.000 | 0.113 | 0.962 | 1.000 | −0.024 | 0.503 | 1.000 | −0.005 | 0.421 | 1.000 | 0.026 | 0.255 | 1.000 |
| 0.01 | 0.098 | 1.042 | 1.000 | 0.113 | 0.962 | 1.000 | −0.024 | 0.503 | 1.000 | −0.005 | 0.421 | 1.000 | 0.026 | 0.255 | 1.000 |
| 0.05 | 0.098 | 1.042 | 1.000 | 0.113 | 0.962 | 1.000 | −0.024 | 0.503 | 1.000 | −0.005 | 0.421 | 1.000 | 0.026 | 0.255 | 1.000 |
| 0.10 | 0.098 | 1.042 | 1.000 | 0.113 | 0.962 | 1.000 | −0.024 | 0.503 | 1.000 | −0.004 | 0.421 | 1.000 | 0.026 | 0.255 | 1.000 |
| 0.20 | 0.098 | 1.042 | 1.000 | 0.113 | 0.962 | 1.000 | −0.024 | 0.503 | 1.000 | −0.004 | 0.421 | 1.000 | 0.026 | 0.255 | 1.000 |
| 0.50 | 0.098 | 1.042 | 1.000 | 0.113 | 0.962 | 1.000 | −0.024 | 0.503 | 1.000 | −0.005 | 0.421 | 1.000 | 0.026 | 0.255 | 1.000 |

Table 156: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon N(0,100)$, $\varepsilon = 0.3$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.01 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.05 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.10 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.20 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.50 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |

Table 157: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon Lo(0,1)$, $\varepsilon = 0$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.004 | 0.226 | 1.000 | 0.025 | 0.132 | 1.000 | −0.010 | 0.088 | 1.000 | −0.010 | 0.068 | 1.000 | 0.002 | 0.049 | 1.000 |
| 0.01 | 0.004 | 0.226 | 1.000 | 0.025 | 0.132 | 1.000 | −0.010 | 0.088 | 1.000 | −0.010 | 0.068 | 1.000 | 0.002 | 0.049 | 1.000 |
| 0.05 | 0.004 | 0.226 | 1.000 | 0.025 | 0.132 | 1.000 | −0.010 | 0.088 | 1.000 | −0.010 | 0.068 | 1.000 | 0.002 | 0.049 | 1.000 |
| 0.10 | 0.004 | 0.226 | 1.000 | 0.025 | 0.132 | 1.000 | −0.010 | 0.088 | 1.000 | −0.010 | 0.068 | 1.000 | 0.002 | 0.049 | 1.000 |
| 0.20 | 0.004 | 0.226 | 1.000 | 0.025 | 0.132 | 1.000 | −0.010 | 0.088 | 1.000 | −0.010 | 0.068 | 1.000 | 0.002 | 0.049 | 1.000 |
| 0.50 | 0.004 | 0.226 | 1.000 | 0.025 | 0.132 | 1.000 | −0.010 | 0.088 | 1.000 | −0.010 | 0.068 | 1.000 | 0.002 | 0.049 | 1.000 |

Table 158: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon Lo(0,1)$, $\varepsilon = 0.01$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.004 | 0.219 | 1.000 | −0.010 | 0.149 | 1.000 | −0.008 | 0.093 | 1.000 | −0.011 | 0.077 | 1.000 | −0.002 | 0.043 | 1.000 |
| 0.01 | −0.004 | 0.219 | 1.000 | −0.010 | 0.149 | 1.000 | −0.008 | 0.093 | 1.000 | −0.011 | 0.077 | 1.000 | −0.002 | 0.043 | 1.000 |
| 0.05 | −0.004 | 0.219 | 1.000 | −0.010 | 0.149 | 1.000 | −0.008 | 0.093 | 1.000 | −0.011 | 0.077 | 1.000 | −0.002 | 0.043 | 1.000 |
| 0.10 | −0.004 | 0.219 | 1.000 | −0.010 | 0.149 | 1.000 | −0.008 | 0.093 | 1.000 | −0.011 | 0.077 | 1.000 | −0.002 | 0.043 | 1.000 |
| 0.20 | −0.004 | 0.219 | 1.000 | −0.010 | 0.149 | 1.000 | −0.008 | 0.093 | 1.000 | −0.011 | 0.077 | 1.000 | −0.002 | 0.043 | 1.000 |
| 0.50 | −0.004 | 0.219 | 1.000 | −0.010 | 0.149 | 1.000 | −0.008 | 0.093 | 1.000 | −0.011 | 0.077 | 1.000 | −0.002 | 0.043 | 1.000 |

Table 159: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon Lo(0,1)$, $\varepsilon = 0.05$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.012 | 0.220 | 1.000 | 0.006 | 0.153 | 1.000 | 0.007 | 0.103 | 1.000 | −0.007 | 0.077 | 1.000 | −0.001 | 0.048 | 1.000 |
| 0.01 | 0.012 | 0.220 | 1.000 | 0.006 | 0.153 | 1.000 | 0.007 | 0.103 | 1.000 | −0.007 | 0.077 | 1.000 | −0.001 | 0.048 | 1.000 |
| 0.05 | 0.012 | 0.220 | 1.000 | 0.006 | 0.153 | 1.000 | 0.007 | 0.103 | 1.000 | −0.007 | 0.077 | 1.000 | −0.001 | 0.048 | 1.000 |
| 0.10 | 0.012 | 0.220 | 1.000 | 0.006 | 0.153 | 1.000 | 0.007 | 0.103 | 1.000 | −0.007 | 0.077 | 1.000 | −0.001 | 0.048 | 1.000 |
| 0.20 | 0.012 | 0.220 | 1.000 | 0.006 | 0.153 | 1.000 | 0.007 | 0.103 | 1.000 | −0.007 | 0.077 | 1.000 | −0.001 | 0.048 | 1.000 |
| 0.50 | 0.012 | 0.220 | 1.000 | 0.006 | 0.153 | 1.000 | 0.007 | 0.103 | 1.000 | −0.007 | 0.077 | 1.000 | −0.001 | 0.048 | 1.000 |

Table 160: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1)+\varepsilon Lo(0,1)$, $\varepsilon = 0.1$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.010 | 0.252 | 1.000 | −0.002 | 0.180 | 1.000 | 0.012 | 0.121 | 1.000 | −0.007 | 0.081 | 1.000 | −0.009 | 0.052 | 1.000 |
| 0.01 | 0.010 | 0.252 | 1.000 | −0.002 | 0.180 | 1.000 | 0.012 | 0.121 | 1.000 | −0.007 | 0.081 | 1.000 | −0.009 | 0.052 | 1.000 |
| 0.05 | 0.010 | 0.252 | 1.000 | −0.002 | 0.180 | 1.000 | 0.012 | 0.121 | 1.000 | −0.007 | 0.081 | 1.000 | −0.009 | 0.052 | 1.000 |
| 0.10 | 0.010 | 0.252 | 1.000 | −0.002 | 0.180 | 1.000 | 0.012 | 0.121 | 1.000 | −0.007 | 0.081 | 1.000 | −0.009 | 0.052 | 1.000 |
| 0.20 | 0.010 | 0.252 | 1.000 | −0.002 | 0.180 | 1.000 | 0.012 | 0.121 | 1.000 | −0.007 | 0.081 | 1.000 | −0.009 | 0.052 | 1.000 |
| 0.50 | 0.010 | 0.252 | 1.000 | −0.002 | 0.180 | 1.000 | 0.012 | 0.121 | 1.000 | −0.007 | 0.081 | 1.000 | −0.009 | 0.052 | 1.000 |

Table 161: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.2$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.018 | 0.286 | 1.000 | 0.024 | 0.181 | 1.000 | −0.018 | 0.114 | 1.000 | −0.010 | 0.091 | 1.000 | −0.006 | 0.060 | 1.000 |
| 0.01 | −0.018 | 0.286 | 1.000 | 0.024 | 0.181 | 1.000 | −0.018 | 0.114 | 1.000 | −0.010 | 0.091 | 1.000 | −0.006 | 0.060 | 1.000 |
| 0.05 | −0.018 | 0.286 | 1.000 | 0.024 | 0.181 | 1.000 | −0.018 | 0.114 | 1.000 | −0.010 | 0.091 | 1.000 | −0.006 | 0.060 | 1.000 |
| 0.10 | −0.018 | 0.286 | 1.000 | 0.024 | 0.181 | 1.000 | −0.018 | 0.114 | 1.000 | −0.010 | 0.091 | 1.000 | −0.006 | 0.060 | 1.000 |
| 0.20 | −0.018 | 0.286 | 1.000 | 0.024 | 0.181 | 1.000 | −0.018 | 0.114 | 1.000 | −0.010 | 0.091 | 1.000 | −0.006 | 0.060 | 1.000 |
| 0.50 | −0.018 | 0.286 | 1.000 | 0.024 | 0.181 | 1.000 | −0.018 | 0.114 | 1.000 | −0.010 | 0.091 | 1.000 | −0.006 | 0.060 | 1.000 |

Table 162: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon Lo(0,1)$, $\varepsilon = 0.3$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.01 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.05 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.10 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.20 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |
| 0.50 | −0.005 | 0.215 | 1.000 | 0.030 | 0.151 | 1.000 | 0.004 | 0.096 | 1.000 | −0.002 | 0.068 | 1.000 | −0.003 | 0.041 | 1.000 |

Table 163: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.001 | 0.231 | 1.000 | 0.024 | 0.132 | 1.000 | 0.000 | 0.203 | 1.000 | 0.016 | 0.263 | 1.000 | 0.002 | 0.068 | 1.000 |
| 0.01 | 0.001 | 0.231 | 1.000 | 0.024 | 0.132 | 1.000 | 0.000 | 0.203 | 1.000 | 0.016 | 0.263 | 1.000 | 0.002 | 0.068 | 1.000 |
| 0.05 | 0.001 | 0.231 | 1.000 | 0.024 | 0.132 | 1.000 | 0.000 | 0.203 | 1.001 | 0.016 | 0.263 | 1.001 | 0.002 | 0.068 | 1.000 |
| 0.10 | 0.001 | 0.231 | 1.000 | 0.024 | 0.132 | 1.000 | 0.000 | 0.203 | 1.000 | 0.016 | 0.263 | 1.001 | 0.002 | 0.068 | 1.000 |
| 0.20 | 0.001 | 0.231 | 1.000 | 0.024 | 0.132 | 1.000 | 0.000 | 0.203 | 0.997 | 0.016 | 0.265 | 0.991 | 0.002 | 0.068 | 1.001 |
| 0.50 | 0.001 | 0.231 | 1.000 | 0.024 | 0.132 | 1.000 | 0.000 | 0.204 | 0.993 | 0.016 | 0.267 | 0.974 | 0.002 | 0.068 | 1.001 |

Table 164: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.01$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.121 | 0.894 | 1.000 | −0.166 | 1.451 | 1.000 | 0.066 | 1.030 | 1.000 | −0.053 | 0.687 | 1.000 | −0.066 | 0.652 | 1.000 |
| 0.01 | −0.121 | 0.894 | 1.000 | −0.166 | 1.452 | 0.999 | 0.067 | 1.031 | 0.998 | −0.053 | 0.687 | 0.999 | −0.066 | 0.651 | 1.003 |
| 0.05 | −0.121 | 0.894 | 1.000 | −0.166 | 1.451 | 1.000 | 0.066 | 1.026 | 1.009 | −0.053 | 0.687 | 1.000 | −0.065 | 0.645 | 1.023 |
| 0.10 | −0.121 | 0.895 | 0.998 | −0.166 | 1.448 | 1.005 | 0.066 | 1.021 | 1.018 | −0.053 | 0.686 | 1.001 | −0.065 | 0.636 | 1.050 |
| 0.20 | −0.121 | 0.895 | 0.998 | −0.165 | 1.440 | 1.015 | 0.066 | 1.021 | 1.018 | −0.053 | 0.682 | 1.014 | −0.064 | 0.627 | 1.080 |
| 0.50 | −0.122 | 0.897 | 0.994 | −0.162 | 1.414 | 1.054 | 0.063 | 0.998 | 1.067 | −0.051 | 0.666 | 1.064 | −0.058 | 0.571 | 1.305 |

Table 165: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.05$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | 0.078 | 2.253 | 1.000 | 0.233 | 2.628 | 1.000 | −0.187 | 3.594 | 1.000 | −0.276 | 1.285 | 1.000 | −0.227 | 1.633 | 1.000 |
| 0.01 | 0.078 | 2.252 | 1.000 | 0.233 | 2.629 | 1.000 | −0.186 | 3.586 | 1.004 | −0.276 | 1.285 | 0.999 | −0.226 | 1.626 | 1.008 |
| 0.05 | 0.078 | 2.251 | 1.001 | 0.231 | 2.616 | 1.009 | −0.182 | 3.541 | 1.030 | −0.275 | 1.280 | 1.008 | −0.222 | 1.596 | 1.047 |
| 0.10 | 0.078 | 2.252 | 1.001 | 0.230 | 2.604 | 1.018 | −0.177 | 3.497 | 1.056 | −0.275 | 1.278 | 1.010 | −0.216 | 1.556 | 1.101 |
| 0.20 | 0.077 | 2.243 | 1.008 | 0.227 | 2.569 | 1.047 | 0.103 | 1.251 | 8.215 | −0.273 | 1.263 | 1.034 | −0.206 | 1.486 | 1.207 |
| 0.50 | 0.076 | 2.233 | 1.018 | 0.025 | 0.733 | 12.95 | 0.100 | 1.234 | 8.445 | −0.266 | 1.218 | 1.110 | −0.125 | 0.946 | 2.988 |

Table 166: Min$\bar{\text{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.1$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.057 | 0.778 | 1.000 | 0.536 | 7.842 | 1.000 | 0.612 | 3.217 | 1.000 | −0.219 | 1.713 | 1.000 | −0.597 | 5.281 | 1.000 |
| 0.01 | −0.057 | 0.778 | 1.000 | 0.534 | 7.823 | 1.005 | 0.612 | 3.212 | 1.003 | −0.219 | 1.714 | 0.999 | −0.586 | 5.175 | 1.042 |
| 0.05 | −0.057 | 0.778 | 1.000 | 0.526 | 7.748 | 1.024 | 0.608 | 3.183 | 1.021 | −0.219 | 1.712 | 1.001 | −0.150 | 1.964 | 7.284 |
| 0.10 | −0.057 | 0.778 | 1.000 | −0.078 | 4.191 | 3.516 | 0.603 | 3.152 | 1.041 | −0.217 | 1.702 | 1.013 | −0.146 | 1.923 | 7.594 |
| 0.20 | −0.057 | 0.777 | 1.001 | −0.076 | 4.155 | 3.576 | 0.391 | 1.890 | 2.881 | −0.216 | 1.688 | 1.030 | −0.140 | 1.849 | 8.211 |
| 0.50 | −0.057 | 0.777 | 1.002 | 0.145 | 3.008 | 6.811 | 0.381 | 1.829 | 3.071 | −0.211 | 1.659 | 1.067 | −0.128 | 1.658 | 10.21 |

Table 167: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.2$, $K = 100$

| $\alpha/n$ | 20 | | | 50 | | | 100 | | | 200 | | | 500 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ | $m(\mu)$ | $s(\mu)$ | $eref(\mu)$ |
| 0.00 | −0.125 | 0.834 | 1.000 | 0.621 | 7.844 | 1.000 | −1.123 | 14.604 | 1.000 | 1.308 | 14.968 | 1.000 | −0.548 | 5.241 | 1.000 |
| 0.01 | −0.125 | 0.834 | 1.000 | 0.620 | 7.837 | 1.002 | 0.249 | 3.554 | 16.91 | −0.097 | 3.790 | 15.71 | −0.536 | 5.135 | 1.042 |
| 0.05 | −0.125 | 0.834 | 1.000 | 0.610 | 7.750 | 1.025 | 0.245 | 3.521 | 17.22 | −0.097 | 3.750 | 16.04 | −0.103 | 1.955 | 7.244 |
| 0.10 | −0.125 | 0.834 | 1.000 | 0.005 | 4.150 | 3.595 | 0.239 | 3.476 | 17.67 | −0.100 | 3.701 | 16.47 | −0.098 | 1.928 | 7.449 |
| 0.20 | −0.125 | 0.834 | 1.000 | 0.003 | 4.102 | 3.680 | −0.009 | 1.858 | 62.11 | −0.263 | 2.978 | 25.26 | −0.089 | 1.872 | 7.903 |
| 0.50 | −0.125 | 0.835 | 0.998 | 0.219 | 2.997 | 6.856 | −0.012 | 1.824 | 64.48 | −0.144 | 2.480 | 36.57 | −0.067 | 1.717 | 9.402 |

Table 168: Min$\bar{\mathrm{D}}_\alpha$: $p = N(0,1)$, data: $(1-\varepsilon)N(0,1) + \varepsilon C(0,1)$, $\varepsilon = 0.3$, $K = 100$

# 5 CONCLUSION

In the examined models, the power superdivergence estimators were equivalent to the common maximum likelihood estimator, except the cases of high contamination by heavy-tailed distribution. Here it displays certain robustness which, however, does not overly impress and which does not compensate the extraordinary computational demands.

The power subdivergence estimators do not possess the very desirable consistency except for the case with escort parameter $\mu = 0$ or $\mu = \bar{\boldsymbol{X}}_n$. These estimates show consistency copying maximum likelihood estimates, and they exhibit high empirical relative efficiency and considerable robustness. This resistance to distant outliers together with consistency and efficiency is a key result of our simulation, and it motivates us to explore the $\max \underline{\mathrm{D}}_\alpha$-estimators escorted by $\mu = \bar{\boldsymbol{X}}_n$ further.

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